# Consensus Domains as Equilibrium Stability Regions of Voting Rules

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**Abstract**

We interpret voting rules as inducing strategic games and study **consensus domains** as subsets of preference profiles with outcomes under specific voting rules. We distinguish **simple consensus** requiring only existence of outcome and **strict consensus** (no profitable coalitional deviation). For unanimity, absolute majority, Condorcet winner, and plurality, we characterize strict consensus domains and prove a strict inclusion hierarchy. Our main result shows that the strict consensus domain of plurality coincides exactly with the set of profiles where the plurality winner is the Condorcet winner. We interpret the Gibbard–Satterthwaite theorem based on the notion of consensus.

Using our consensus definitions we define "plural dictatorship" as a set of preference profiles, and offer an actionable observation: *In plural dictatorship, finding the Condorcet consensus (either informally as it happened in Hungary before the 2026 parliamentary elections, or semiformally with an activist-led preelection vote), and tactically voting to the Condorcet winner can turn the plural dictatorship into democracy.*

## 1 Introduction

A voting rule aggregates preferences into a social choice. When voters are strategic, each rule induces a game. A natural question is: **for which preference profiles is the outcome stable against manipulation?** We formalize this via *consensus domains*, defined relative to equilibrium concepts.

Our contribution:

1. Define simple and strict consensus domains for voting rules as equilibrium stability regions.
2. Characterize strict consensus domains for unanimity, majority, Condorcet, and plurality.
3. Establish a strict inclusion hierarchy and show that **plurality's strict domain equals its intersection with the Condorcet domain**.
4. Define plural dictatorship based on consensus domains, and offer an actionable insight to turn it into democracy

# 2 Model

## 2.1 Preferences and Profiles

Let $N = \{1, \dots, n\}$ be voters, $X$ a finite set of alternatives ($|X| \geq 3$). Each voter $i$ has a strict preference $\succ_i$. A profile is $p = (\succ_i)_{i \in N}$.

## 2.2 Voting Rules

A (possibly partial) rule is $F : \mathcal{P} \to X \cup \{\varnothing\}$.

We study:

- **Unanimity**: selects $x$ if all voters rank $x$ first.
- **Absolute majority**: selects $x$ if it has $> n/2$ first-place votes.
- **Condorcet rule**: selects the Condorcet winner if it exists.
- **Plurality**: selects the alternative with the most first-place votes.

# 3 Strategic Form and Equilibria

## 3.1 Induced Game

Given $F$, each voter reports a ranking (or equivalently, a ballot). Let $\mathcal{B}_i$ be admissible ballots (e.g., linear orders). The induced normal-form game is:

$$G(F, p) = \big(N, (\mathcal{B}_i)_{i \in N}, (u_i)_{i \in N}\big),$$

where outcomes are $F(b)$ and utilities $u_i$ represent $\succ_i$.

## 3.2 Equilibrium Notions

- **Nash equilibrium (NE)**: no unilateral profitable deviation.
- **Coalition-proof equilibrium (CPE)** (Bernheim–Peleg–Whinston): no coalition can deviate to a profile that makes all its members weakly better off and at least one strictly better off, with internal stability against further deviations.

We use these as stability notions for defining consensus domains.

# 4 Consensus Domains

**Definition 1** (simple consensus domain)**.** *A profile $p$ lies in the simple consensus domain of $F$ if truthful reporting yields a (non-null) winner.*

**Definition 2** (strict consensus domain)**.** *A profile $p$ lies in the strict consensus domain of $F$ if truthful reporting is a coalition-proof equilibrium of $G(F, p)$ yielding a (non-null) winner.*

## 5 Basic Facts

**Lemma 1** (Majority ⇒ Condorcet)**.** *If $x$ has $> n/2$ first-place votes, then $x$ beats every $y \neq x$ pairwise, hence is the Condorcet winner.*

*Proof.* The majority winner pairwise beats all other candidates. □

## 6 Strict Consensus: Characterization

**Proposition 1** (Unanimity)**.** *Unanimity profiles lie in the strict-consensus domain of all four rules.*

*Proof.* No deviation can improve any coalition. □

**Proposition 2** (Absolute Majority)**.** *Profiles with an absolute majority winner lie in the strict-consensus domains of plurality and Condorcet.*

*Proof.* Let $x$ have $> n/2$ first-place votes. Any deviation that overturns $x$ must move more than half the electorate; those voters already rank $x$ top, so cannot be (weakly) better off. □

**Theorem 1** (Strict plurality consensus + Condorcet implies Condorcet outcome)**.** *Let $F$ be the pluralitthe plurality voting rule and let $p$ be a preference profile. If*

- *$p \in SC_F$ (i.e. $p$ lies in the strict-consensus domain of $F$, meaning truthful reporting is coalition-proof under $F$), and*
- *$p \in CW$ (i.e. a Condorcet winner exists at $p$),*

*then*

$$F(p) = c,$$

*where $c$ is the Condorcet winner.*

*Proof.* Let $c$ denote the Condorcet winner at profile $p$, and suppose for contradiction that

$$F(p) = a \neq c.$$

Since $c$ is a Condorcet winner, we have:

$$|\{i \in N : c \succ_i a\}| > |\{i \in N : a \succ_i c\}|.$$

Define the coalition

$$S = \{i \in N : c \succ_i a\}.$$

Then every voter in $S$ strictly prefers $c$ to $a$, and $S$ is strictly larger than the opposing set.

Now consider a coordinated deviation by coalition $S$ in which members modify their ballots in a way that maximally supports $c$ relative to $a$ (e.g., ranking $c$ first or otherwise optimally promoting $c$ under rule $F$).

Because all members of $S$ strictly prefer $c \succ_i a$, any outcome change from $a$ to $c$ is strictly beneficial for all $i \in S$, and beneficial for at least one.

Moreover, since $S$ constitutes a strict majority over the set favoring $a$ against $c$, coalition $S$ has sufficient mass to alter the relative standing of $c$ versus $a$ under any rule where outcomes respond to aggregated support.

Thus, the deviation yields an outcome $c$ that is strictly preferred by all members of $S$, contradicting the assumption that $p \in SC_F$ (coalition-proofness).

Hence the assumption $F(p) \neq c$ is impossible, and we conclude:

$$F(p) = c.$$

□

**Theorem 2** (Plurality instability off Condorcet)**.** *If a Condorcet winner c exists and differs from the plurality winner a, then truthful play is not coalition-proof under plurality.*

*Proof.* Let $S = \{i : c \succ_i a\}$. Since $c$ is Condorcet, $|S| > |\{i : a \succ_i c\}|$. Consider a joint deviation where all $i \in S$ rank $c$ first. Then $c$ obtains at least $|S|$ first-place votes, while $a$ obtains at most $|\{i : a \succ_i c\}|$, hence $c$ becomes the plurality winner. Every $i \in S$ strictly prefers $c$ to $a$, so the deviation is profitable for all members. □

**Corollary 1.** *The strict-consensus domain of plurality is contained in the Condorcet domain.*

**Theorem 3** (Characterization of strict plurality consensus)**.** *A profile lies in the strict-consensus domain of plurality* ***iff*** *the plurality winner is the Condorcet winner.*

*Proof.* (⇒) Follows from Theorem 2.
(⇐) Follows from Theorem 1. (Let $x$ be both plurality and Condorcet winner. Suppose a coalition deviation yields $y \neq x$. Since $x$ beats $y$ pairwise, a strict majority prefers $x$ to $y$; hence no coalition making all members weakly better off can exist.) □

# 7 Inclusion Hierarchy

**Theorem 4** (Strict inclusion chain)**.**

$$\textit{Unanimity} \subset \textit{Absolute Majority} \subset \textit{Strict Plurality} \subset \textit{Condorcet},$$

*with all inclusions strict.*

*Proof.*

- Unanimity ⊂ Majority: trivial.
- Majority ⊂ Strict Plurality: by Lemma 1 and Theorem 2.

- Strict Plurality $\subset$ Condorcet: by Corollary 1.

Strictness follows from standard examples: (i) Condorcet without majority; (ii) plurality–Condorcet coincidence without majority; etc. □

**Conjecture 1.** *There is a weakest strict consensus, and it is the Condorcet consensus.*

I did not prove it yet, but given that the proof of the Gibbard-Shattertwaite theorem uses Condorcet cycles, it seems provable.

**Observation 1** (For any deterministic voting rule with more than two outcomes, there is a domain where there is no strict consensus.)**.**

This is just the rewording of the Gibbard-Shattertwaite theorem using our consensus notion.

**Corollary 2** (No point of looking for consensus using voting where there isn't one)**.**

This trivially follows from the above. When there is no consensus, one should be forged by discussions.

**Definition 3** (Strict plural dictatorship)**.** *Plural dictatorship* $=$ *Condorcet agreement*$\setminus$ *Plurality agreement*,

Plural dictatorship is the rule of the few ($\{i : a \succ_i c\}$) over the many ($\{j : c \succ_j a\}$). Where $c$ is the Condorcet winner, and $a$ is the honest plurality winner.

**Observation 2** (Omissional plural dictatorship)**.**

We can talk about omissional plural dictatorship as well. It happens in a system with repeated plural voting, when voters eliminate all candidates using tactical voting such that the ballot only contains candidates which are below the "none of above" candidate of the original candidate set.

A formal definition of omissional plural dictatorship needs a model which considers shift of platform for candidates and voters - so a discussion of Duverger's law in a framework considering both candidate and voter behaviour is possible.

Given the different forms of plural dictatorship, the following questions need further research:

- how and when the plural rule results in simple and omissional plural dictatorship, and
- are there cases when the plural rule does not lead to one of the above kind of dictatorships, and what are the conditions for them?

Probably the second question is moot, as we can use voting rules not leading to dictatorship.

**Observation 3** (Plural dictatorship can be overthrown by coordination)**.**

It is an interpretation of Theorem 2.

This happened in Hungary in the 2026 parliamentary elections. In this case the Condorcet winner was determined using informal ways. But an activist-led preelection can determine the Condorcet winner, and the voters can coordinate accordingly. Note that this is a mathematical result, with conditions. In 2022 the Hungarian public did make a preelection and its winner was the Condorcet winner among the voters of the preelection. But the winner was not the Condorcet winner among all woters of the real election (probably due to lack of coordination and bad campaign strategy).

## 8 Simple Consensus vs Strict Consensus

The above hierarchy is specific to **coalitional** stability. Under **simple consensus**, the structure changes:

- By the Gibbard–Satterthwaite theorem, any non-dictatorial, onto rule is manipulable somewhere; thus simple consensus domains are necessarily restricted.
- For plurality, there exist profiles where truthful voting is a Nash equilibrium even when the winner is not Condorcet (e.g., due to coordination failures or pivotality considerations).
- Hence, unlike strict consensus, **simple consensus domains are not nested by Condorcet dominance**.

**Takeaway.** Coalition-proofness aligns plurality with Condorcet, while unilateral stability does not.

## 9 Discussion

- **Geometric view.** Each rule induces a "stability region" in preference space. Condorcet is largest among the four considered; plurality's strict region is exactly its intersection with Condorcet.
- **Mechanism insight.** Plurality fails coalition-proofness precisely when a Condorcet-majority coalition can coordinate on a single alternative.
- **Design implication.** If coalition-proofness is desired, plurality inherits Condorcet consistency on its stable domain.

## 10 Conclusion

We formalized consensus domains as equilibrium stability regions and proved that, under coalition-proofness, plurality's stable domain collapses to the Condorcet-consistent region. The clean nesting contrasts sharply with simple consensus, reflecting classic impossibility results and the limits of unilateral stability.

We also found ideological results based on social choice theory proofs:

- The notion of simple and omissional plural dictatorship
- The way to overthrow simple plural dictatorship
- The observation that when there is no consensus, there is no point for looking for one using vote, but one should be forged through discussions.

Further research is needed into the properties of simple and omissional plural dictatorship, and the ways to prevent and overthrow them.

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