The Wilson letters from his floppy

file at A7402

13 July 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following thirty-some odd sequences, most to the first 101 terms for inclusion in your second edition of the above. "It is easy to prove that for any given natural number k the equation I (n+k) = I (n) has at least one solution in the natural numbers n." page 231. There-fore the following sequence: 1, 4, 3, 8, 5, 24, 5, 13, 9, 20, 7, 48, 13, 16, 13, 26, 17, 52, 19, 37, 21, 44, 13, 96, 25, 34, 27, 32, 13, 124, 17, 52, 33, 41, 19, 104, 35, 52, 37, 65, 25, 123, 17, 73, 39, 92, 41, 183, 35, 76, 39, 68, 53, 156, 35, 64, 57, 116, 41, 248, 61, 73, 61, 104, 65, 144, 67, 82, 41, 140, 37, 208, 73, 124, 65, 104, 37, 267, 65, 109, 81, 143, 83, 241, 85, 148, 87, 143, 37, 365, 41, 184, 61, 188, 55, 219, 97, 97, 91, 152, 101, ...
" ... have proved that for every natural number k<2*10^58 the equation I (n+k) = I (n) has at least two solutions in natural numbers n." page 232. Therefore the following sequence: 3, 7, 5, 14, 9, 34, 7, 16, 15, 26, 11, 68, 39, 28, 15, 32, 33, 72, 25, 40, 35, 56, 17, 101, 45, 37, 45, 56, 29, 152, 31, 61, 39, 56, 35, 144, 37, 61, 39, 74, 41, 128, 35, 88, 45, 161, 47, 192, 49, 82, 51, 74, 95, 216, 43, 97, 75, 203, 59, 304, 91, 88, 63, 122, 117, 194, 129, 112, 51, 146, 71, 288, 117, 148, 73, 119, 55, 292, 73, 130, 135, 146, 225, 246, 133, 172, 95, 146, 89, 372, 65, 259, 93, 194, 89, 339, 123, 112, 99, 164, 143, ...
The difference between the above two series is the following sequence:

2, 3, 2, 6, 4, 10, 2, 3, 6, 6, 4, 20, 26, 12, 2, 6, 16, 20, 6, 3, 14, 12, 4, 5, 20, 3, 18, 24, 16, 28, 14, 9, 6, 15, 16, 40, 2, 9, 2, 9, 16, 5, 18, 15, 6, 69, 6, 9, 14, 6, 12, 6, 42, 60, 8, 33, 18, 87, 18, 56, 30, 15, 2, 18, 52, 50, 62, 30, 10, 6, 34, 80, 44, 24, 8, 15, 18, 25, 8, 21, 54, 3, 142, 5, 48, 24, 8, 3, 52, 7, 24, 75, 32, 6, 34, 120, 26, 15, 8, 12, 42, ...

As long as we are on this train of thought, then the next logical

sequence is the third occurrence, and it is as follows: 15, 8, (3540000),

16, 15, 36, 21, 19, (100000), 35, 27, 72, 51, 34, 17, 38, 35, 73, 57, 52, (100000), 73, 23, 109, 75, 52, 55, 68, 51, 180, 39, 64, 45, 68, 75, 146, 49, 64, 45, 80, 111, 148, 43, 91, 51, 182, 65, 202, 147, 100, 57, 104, 123, 219, 55, 112, 91, 232, 177, 325, 93, 109, 105, 128, 183, 219, 201, 136, 57, 152, 111, 292, 175, 238, 75, 122, 77, 312, 79, 148, 145, 152, 243, 256, 153, 194, 99, 176, 119, 386, 91, 322, 135, 329, 95, 366, 273, 178, 117, 185, 237, .. .
Continuing the next sequence is the fourth occurrence, and it is as

follows: 104, 10, (3540000), 20, 21, 39, 45, 25, (100000), 100, 33, 78, 63,

41, 21, 50, 39, 82, 225, 55, (100000), 77, 69, 111, 99, 89, (100000), 82, 87, 194, 93, 76, 55, 74, 105, 164, 111, 73, 65, 95, 123, 153, 85, 112, 55, 184, 77, 218, 315, 130, 63, 178, 159, 246, 91, 133, 95, 266, 357, 360, 183, 124, 115, 152, 195, 244, 429, 148, 69, 155, 153, 303, 219, 259, 85, 128, 123, 327, 111, 157, 165, 164, 249, 296, 195, 247, 135, 182, 155, 456, 273, 353, 155, 365, 171, 369, 291, 181, 135, 200, 267, ...
And the final sequence is the fifth occurrence, and it is as follows:

164, 26, (3540000), 35, 15556, 43, 75, 28, (100000), 130, 45, 86, 75, 56, (100000), 56, 51, 102, 273, 70, (100000), 80, 99, 136, 105, 91, (100000), 112, 93, 208, 117, 100, (50000), 119, 111, 181, 399, 76, 105, 104, 153, 168, 129, 146, 63, 200, 135, 222, 525, 175, 85, 182, 429, 268, 99, 136, (50000), 290, 429, 369, 207, 190, (50000), 200, 255, 264, 441, 169, 115, 170, 213, 327, 651, 281, 105, 146, 125, 372, 231, 160, (50000), 224, 261, 306, 219, 286, 145, 185, 267, 482, 357, 364, 195, 376, 285, 384, 385, 193, 165, 260, 303, ...
In the same vein but a different function 'Sum of the Divisors' the

following sequence is the first occurrence for which o (n+k)= o (n): 14, 33,

382, 51, 6, 20, 10, 15, 14, 21, 28, 35, 182, 24, 26, 30, 142, 40, 34, 42, 20, 57, 135, 70, 30, 99, 42, 66, 406, 88, 56, 60, 54, 93, 24, 105, 248, 147, 44, 63, 30, 80, 435, 114, 52, 196, 310, 140, 40, 105, 92, 160, 66, 120, 140, 105, 88, 352, 154, 224, 118, 177, 60, 117, 78, 220, 182, 135, 8786, 96, 112, 210, 752, 135, 92, 294, 110, 365, 735, 126, 126, 204, 60, 270, 102, 105, 254, 165, 78, 264, 88, 195, 174, 440, 114, 280, 138, 168, 124, 210, 316, ...
For the second occurrance the following sequence: 206, 54, 11935, 66,

46, 155, 62, 69, 16, 174, 154, 104, 782, 33, 62, 55, 238, 60, 158, 51, 30, 85, 231, 143, 46, 150, 48, 159, 496, 161, 58, 110, 35562, 96, 42, 130, 302, 246, 56, 84, 54, 135, 602, 123, 70, 205, 658, 165, 66, 132, 158, 198, 406, 180, 166, 132, 102, 852, 376, 315, 188, 224, 76, 120, 526, 232, 795, 186, 24885, 120, 945, 260, 862, 280, 130, 352, 190, 459, 1034, 147, 144, 748, 184, 370, 166, 390, 358, 228, 114, 352, 130, 267, 11842, 736, 170, 330, 686, 231, 154, 255, 658, ...
And the difference between the two produces the following sequence:

192, 21, 11553, 15, 40, 135, 52, 54, 2, 153, 126, 69, 600, 9, 36, 25, 96, 20, 124, 9, 10, 28, 96, 73, 16, 51, 6, 93, 90, 73, 2, 50, 35508, 3, 18, 25, 54, 99, 12, 21, 24, 55, 167, 9, 18, 9, 348, 25, 26, 27, 66, 38, 340, 60, 26, 27, 14, 500, 222, 91, 70, 47, 16, 3, 448, 12, 613, 51, 16099, 24, 833, 50, 110, 145, 38, 58, 80, 94, 299, 21, 18, 544, 124, 100, 64, 285, 104, 63, 36, 88, 42, 72, 11668, 296, 56, 50, 548, 63, 30, 45, 342, ...
On page 234, "for every natural number s there exists a natural number m such that the equation I (n) = m has precisely s solutions in natural numbers. We do not know the answer to this question even in the simple case of s=1. ... As was show by V. L. Klee Jr. [3], there are no such numbers m < 10^400." Restating the original series and then continuing it, I present: "1, 2, 4, 8, 12, 32, 36, 40, 24, 48, 160, 396, 2268, 704,"
312, 72, 336, 216, 936, 144, 624, 1056, 1760, 360, 2560, 384, 288, 1320, 3696, 240, 768, 9000, 432, 7128, 4200, 480, 576, 1296, 1200, 15936, 3312, 3072, 3240, 864, 3120, 7344, 3888, 7220, 1680, 4992, 17640, 2016, 1152, 6000, 12288, 4752, 2688, 3024, 13680, 9984, 1728, 1920, 2400, 7560, 2304, 22848, 8400, 29160, 5376, 3360, 1440, 13248, 11040, 27720, 21840, 9072,

38640, 9360, 81216, 4032, 5280, 4800, 4608, 16896, 3456, 3840, 10800, 9504, 18000, 23520, 39936, 5040, 26208, 27360, 6480, 9216, 2880, 26496, 34272, 23328, 28080, ...

However, just as there are solutions for I (n) = m, so are there values of m to which s = 0 or restated, there are no solutions in m. Beiler, page 91. They begin: 14, 26, 34, 38, 50, 62, 68, 74, 76, 86, 90, 94, 98, 114, 118, 122, 124, 134, 142, 146, 152, 154, 158, 170, 174, 182, 186, 188, 194, 202, 206, 214, 218, 230, 234, 236, 242, 244, 246, 248, 254, 258, 266, 274, 278, 284, 286, 290, 298, 302, 304, 308, 314, 318, 322, 326, 334, 338, 340, 350, 354, 362, 364, 370, 374, 376, 386, 390, 394, 398, 402, 404, 406, 410, 412, 414, 422, 426, 428, 434, 436, 446, 450, 454, 458, 470, 472, 474, 482, 484, 488, 494, 496, 510, 514, 516, 518, 526, 530, 532, 534, ...

Or if you like the above divided by two so as to save some room: 7, 13,

17, 19, 25, 31, 34, 37, 38, 43, 45, 47, 49, 57, 59, 61, 62, 67, 71, 73, 76, 77, 79, 85, 87, 91, 93, 94, 97, 101, 103, 107, 109, 115, 117, 118, 121, 122, 123, 124, 127, 129, 133, 137, 139, 142, 143, 145, 149, 151, 152, 154, 157, 159, 161, 163, 167, 169, 170, 175, 177, 181, 182, 185, 187, 188, 193, 195, 197, 199, 201, 202, 203, 205, 206, 207, 211, 213, 214, 217, 218, 223, 225, 227, 229, 235, 236, 237, 241, 242, 244, 247, 248, 255, 257, 258, 259, 263, 265, 266, 267, 269, ...

The equation I (n) = m has just two solutions: 1, 10, 22, 28, 30, 46,

52, 54, 58, 66, 70, 78, 82, 102, 106, 110, 126, 130, 136, 138, 148, 150, 166, 172, 178, 190, 196, 198, 210, 222, 226, 228, 238, 250, 262, 268, 270, 282, 292, 294, 306, 310, 316, 330, 342, 346, 358, 366, 372, 378, 382, 388, 418, 430, 438, 442, 462, 466, 478, 490, 498, 502, 506, 508, 522, 546, 556, 562, 568, 570, 580, 586, 598, 606, 618, 630, 642, 646, 652, 658, 676, 682, 690, 708, 718, 726, 738, 742, 750, 772, 786, 796, 808, 810, 812, 822, 826, 838, 852, 856, 858, ...

The equation I (n) = m has just three solutions: 2, 44, 56, 92, 104,

116, 140, 164, 204, 212, 260, 296, 332, 344, 356, 380, 392, 444, 452, 476, 524, 536, 564, 584, 588, 620, 632, 684, 692, 716, 744, 764, 776, 836, 860, 884, 932, 956, 980, 1004, 1016, 1112, 1124, 1136, 1172, 1196, 1284, 1292, 1304, 1316, 1352, 1364, 1416, 1436, 1484, 1544, 1592, 1616, 1644, 1652, 1676, 1704, 1712, 1724, 1772, 1812, 1820, 1880, 1892, 1940, 1952, 1964, 2036, 2060, 2124, 2172, 2180, 2192, 2204, 2216, 2288, 2300, 2324, 2360, 2372, 2384, 2432, 2444, 2456, 2516, 2564, 2604, 2612, 2636, 2732, 2744, 2844, 2852, 2876, 2892, 2900, ...

The equation I (n) = m has just four solutions: 4, 6, 18, 42, 100, 162,

184, 208, 328, 424, 460, 468, 486, 492, 616, 636, 664, 688, 700, 712, 784, 820, 900, 904, 1020, 1060, 1072, 1168, 1240, 1264, 1276, 1288, 1300, 1356, 1360, 1384, 1404, 1458, 1480, 1528, 1672, 1740, 1768, 1864, 1896, 1900, 1908, 2008, 2028, 2032, 2148, 2196, 2220, 2224, 2248, 2296, 2328, 2332, 2344, 2380, 2500, 2508, 2568, 2584, 2620, 2628, 2704, 2860, 2868, 2872, 3012, 3180, 3184, 3204, 3220, 3232, 3256, 3288, 3304, 3352, 3424, 3460, 3544, 3580, 3624, 3820, 3904, 3912, 3916, 3948, 4068, 4120, 4180, 4308, 4344, 4360, 4384, 4420, 4422, 4432, 4632, ...

The equation I (n) = m has just five solutions: 8, 20, 220, 272, 300,

368, 416, 456, 500, 656, 732, 848, 876, 1092, 1160, 1212, 1236, 1328, 1376, 1424, 1568, 1624, 1716, 1808, 2144, 2244, 2336, 2420, 2460, 2480, 2528, 2556, 2768, 3056, 3080, 3252, 3320, 3344, 3536, 3560, 3612, 3728, 3732, 3900, 4016, 4020, 4064, 4260, 4448, 4496, 4520, 4688, 4692, 5100, 5168, 5232, 5340, 5360, 5408, 5512, 5744, 5840, 5984, 6036, 6132, 6156, 6200, 6320, 6368, 6380, 6464, 6608, 6636, 6704, 6848, 7088, 7212, 7248, 7536, 7700, 7808, 7932, 8004, 8120, 8240, 8600, 8720, 8768, 8864, 9012, 9276, 9320, 9488, 9536, 9560, 9728, 9800, 9824, 9940, ...

On page 235, "It is not known whether for every natural number k there exists a natural number m for which the equation o (x) = m has precisely k solutions in natural numbers x. This follows from the conjecture H (...). It can be proved that if m denotes the least of the numbers for which o (x) = m has precisely k solutions, then" : "1, 12, 24, 96, 72, 168, 240, 432, 360, 504, 576, 1512, 1080, 1008, 720, 2304, 3600, 5376, 2160, 1440," But this quoted series is incorrect. The problem with the above sequence is not that 432 does not have eight solutions for o (n) = 432 (they being 230, 238, 255, 321, 355, 371, 391 & 431.), but that 432 is not the first number to possess this trait. The number 336 is, with the eight solutions being 132, 140, 182, 188, 195, 249, 287 & 299. Also 2520 is the number with 19 solutions and both 2160 and 1440 have one more solution that they are credited with, although they still retain the distinction of being the first. (2160 has as its 20 solutions: 870, 918, 920, 952, 1074, 1246, 1298, 1334, 1335, 1431, 1438, 1479, 1595, 1615, 1795, 1883, 1969, 2033, 2047 & 2059.) (1440 has as its 21 solutions: 552, 570, 594, 616, 790, 826, 874, 885, 957, 958, 969, 1015, 1045, 1077, 1195, 1253, 1343, 1349, 1357, 1363 & 1439.) Let me restate the series correctly from the beginning:

1, 12, 24, 96, 72, 168, 240, 336, 360, 504, 576, 1512, 1080, 1008, 720, 2304, 3600, 5376, 2520, 2160, 1440, 10416, 13392, 3360, 4032, 3024, 7056, 6720, 2880, 6480, 10800, 13104, 5040, 6048, 4320, 13440, 5760, 18720, 20736, 19152, 22680, 43680, 28080, 26208, 14400, 16128, 25200, 11520, 8640, 78120, 18144, 21600, 62208, 35280, 97200, 62496, 142848, 10080, 15120, 55440, 44640, 66960, 38880, 24192, 42336, 98496, 52416, 17280, 97920, 64512, 46080, 63360, 123120, 25920, 54720, 117936, 231840, 45360, 20160, 127680, 57600, 43200, 75600, 200880, 48384, 228096, 158400, 147840, 131328, 215040, 334800, 275184, 172368, 196992, 133920, 142560, 34560, 30240, 368640, 72576, (392212), ...

However, just as there are solutions for o (n) = m, so are there values of m to which s = 0 or restated, there are no solutions in m. They begin:

2, 5, 9, 10, 11, 16, 17, 19, 21, 22, 23, 25, 26, 27, 29, 33, 34, 35, 37, 41, 43, 45, 46, 47, 49, 50, 51, 52, 53, 55, 58, 59, 61, 64, 65, 66, 67, 69, 70, 71, 73, 75, 76, 77, 79, 81, 82, 83, 85, 86, 87, 88, 89, 92, 94, 95, 97, 99, 100, 101, 103, 105, 106, 107, 109, 111, 113, 115, 116, 117, 118, 119, 122, 123, 125, 129, 130, 131, 134, 135, 136, 137, 139, 141, 142, 143, 145, 146, 147, 148, 149, 151, 153, 154, 155, 157, 159, 161, 163, 165, 166, ...

The equation o (n) = m has just one solutions: 1, 3, 4, 6, 7, 8, 13,

14, 15, 20, 28, 30, 36, 38, 39, 40, 44, 57, 62, 63, 68, 74, 78, 91, 93, 102, 110, 112, 121, 127, 133, 138, 150, 158, 160, 162, 164, 171, 174, 176, 183, 194, 195, 198, 200, 204, 212, 217, 222, 230, 242, 255, 256, 258, 260, 266, 278, 282, 284, 296, 300, 304, 306, 307, 314, 318, 330, 332, 338, 348, 350, 352, 354, 363, 364, 368, 374, 380, 381, 396, 398, 400, 402, 410, 414, 422, 458, 462, 464, 465, 474, 476, 488, 494, 496, 500, 508, 510, 511, 512, 518, ...

The equation o (n) = m has just two solutions: 12, 18, 31, 32, 54, 56,

80, 98, 104, 108, 114, 124, 126, 128, 132, 140, 152, 156, 182, 186, 210, 264, 272, 280, 308, 320, 342, 378, 390, 392, 399, 403, 408, 416, 440, 444, 448, 492, 522, 532, 570, 572, 594, 608, 630, 632, 726, 762, 770, 774, 780, 784, 800, 828, 868, 880, 884, 900, 920, 924, 942, 948, 954, 984, 1014, 1024, 1026, 1032, 1040, 1044, 1062, 1088, 1098, 1110, 1164, 1178, 1188, 1194, 1218, 1230, 1272, 1280, 1328, 1350, 1352, 1364, 1374, 1386, 1408, 1428, 1430, 1472, 1484, 1500, 1520, 1568, 1572, 1608, 1610, 1656, 1664, ...

The equation o (n) = m has just three solutions: 24, 42, 48, 60, 84,

90, 224, 228, 234, 248, 270, 294, 324, 450, 468, 528, 558, 620, 640, 660, 810, 882, 888, 896, 968, 972, 1020, 1050, 1104, 1116, 1140, 1216, 1232, 1240, 1274, 1332, 1392, 1400, 1452, 1456, 1464, 1482, 1524, 1530, 1600, 1694, 1716, 1760, 1890, 1896, 1932, 1960, 1968, 2028, 2128, 2176, 2256, 2286, 2294, 2418, 2436, 2460, 2464, 2484, 2660, 2772, 2964, 3042, 3132, 3280, 3294, 3328, 3384, 3408, 3584, 3684, 3724, 3808, 3852, 3864, 3876, 3912, 3924, 3948, 3984, 3990, 4160, 4230, 4248, 4260, 4290, 4298, 4312, 4446, 4452, 4488, 4576, 4776, 4824, 4944, 4968, ...

The equation o (n) = m has just four solutions: 96, 120, 180, 312, 372,

420, 434, 456, 540, 546, 560, 624, 702, 728, 798, 816, 930, 1064, 1120, 1170, 1404, 1632, 1638, 1674, 1710, 1776, 1792, 1944, 2100, 2240, 2544, 2560, 2664, 2760, 2800, 2844, 2856, 2940, 2952, 3000, 3040, 3048, 3060, 3080, 3096, 3108, 3224, 3432, 3492, 3510, 3564, 3768, 3822, 3920, 4004, 4140, 4356, 4424, 4572, 4644, 4650, 4656, 4712, 4836, 4914, 5004, 5088, 5120, 5130, 5320, 5496, 5568, 5640, 5652, 5670, 5724, 5832, 6200, 6288, 6400, 6510, 6672, 6776, 6858, 6960, 7224, 7280, 7360, 7448, 7524, 7536, 7650, 7688, 7704, 7872, 7944, 7968, 8060, 8244, 8256, 8460, ...

The equation o (n) = m has just five solutions: 72, 144, 192, 216, 588,

600, 648, 792, 936, 992, 1056, 1224, 1302, 1320, 1560, 1736, 1980, 2040, 2088, 2112, 2268, 2448, 2730, 2790, 2912, 3038, 3136, 3312, 3472, 3520, 3534, 3552, 3672, 3792, 3816, 3936, 4056, 4092, 4340, 4440, 4864, 4872, 4920, 4960, 5082, 5334, 5600, 5796, 5904, 5940, 6096, 6156, 6768, 6936, 7168, 7368, 7380, 7800, 7936, 8148, 8280, 8320, 8432, 8580, 8664, 8704, 8856, 8904, 9180, 9312, 9432, 9552, 9648, 9660, 9768, 9900, 9920, 10032, 10200, 10240, 10248, 10320, 10530, 10602, 10692, 10980, 10992, 11016, 11136, 11256, 11400, 11440, 11700, 11844, 11928, 12012, 12152, 12192, 12264, 12400, 12648, ...

The following is the first occurrence for n when I (n) = k/2 inwhich n is not a prime one less than n: 4, 8, 9, 15, 22, 21, 0, 32, 27, 25, 46, 35, 0, 58, 62, 51, 0, 57, 0, 55, 49, 69, 94, 65, 0, 106, 81, 87, 118, 77, 0, 85, 134, 0, 142, 91, 0, 0, 158, 123, 166, 129, 0, 115, 0, 141, 0, 119, 0, 125, 206, 159, 214, 133, 121, 145, 0, 177, 0, 143, 0, 0, 254, 255, 262, 161, 0, 274, 278, 213, 0, 185, 0, 298, 302, 0, 0, 169, 0, 187, 243, 249, 334, 203, 0, 346, 0, 267, 358, 209, 0, 235, 0, 0, 382, 221, 0, 394, 398, 275, 0, ...

The following is the last occurrence for n when I (n) = k/2: 2, 6, 12, 18, 30, 22, 42, 0, 60, 54, 66, 46, 90, 0, 58, 62, 120, 0, 126, 0, 150, 98, 138, 94, 210, 0, 106, 162, 174, 118, 198, 0, 240, 134, 0, 142, 270, 0, 0, 158, 330, 166, 294, 0, 276, 0, 282, 0, 420, 0, 250, 206, 318, 214, 378, 242, 348, 0, 354, 0, 462, 0, 0, 254, 510, 262, 414, 0, 274, 278, 426, 0, 630, 0, 298, 302, 0, 0, 474, 0, 660, 486, 498, 334, 588, 0, 346, 0, 690, 358, 594, 0, 564, 0, 0, 382, 840, 0, 394, 398, 750, 0, ...

(In the preceeding two sequences, the series of occurrences of Zeroes matches an earlier sequence presented at the bottom of page 3.)

The following is the first occurrence for n when o (n) = k: 1, 0, 2, 3, 0, 5, 4, 7, 0, 0, 0, 6, 9, 13, 8, 0, 0, 10, 0, 19, 0, 0, 0, 14, 0, 0, 0, 12, 0, 29, 16, 21, 0, 0, 0, 22, 0, 37, 18, 27, 0, 20, 0, 43, 0, 0, 0, 33, 0, 0, 0, 0, 0, 34, 0, 28, 49, 0, 0, 24, 0, 61, 32, 0, 0, 0, 0, 67, 0, 0, 0, 30, 0, 73, 0, 0, 0, 45, 0, 57, 0, 0, 0, 44, 0, 0, 0, 0, 0, 40, 36, 0,

50, 0, 0, 42, 0, 52, 0, 0, 0, ...
The following is the first occurrence for n when o (n) = k less the Zeroes:
1, 2, 3, 5, 4, 7, 6, 9, 13, 8, 10, 19, 14, 12, 29, 16, 21, 22, 37,
18, 27, 20, 43, 33, 34, 28, 49, 24, 61, 32, 67, 30, 73, 45, 57, 44, 40,
36, 50, 42, 52, 101, 63, 85, 109, 91, 74, 54, 81, 48, 68, 64, 93, 86, 121,
137, 76, 66, 149, 111, 99, 157, 133, 106, 163, 60, 98, 173, 129, 88, 117,
169, 80, 105, 193, 72, 197, 199, 134, 104, 211, 102, 100, 146, 84, 147,
229, 90, 114, 241, 112, 96, 128, 217, 257, 171, 215, 148, 136, 201, 277,
... .
To expand on your Sequence Nbr. 1215, I (n) = I (n+1) : "1, 3, 15, 104,
164, 194, 255, 495, 584, 975, 2204, 2625, 2834, 3255, 3705, 5186, 5187,"
10604, 11714, 13365, 18315, 22935, 25545, 32864, 38804, 39524, 46215, 48704,
49215, 49335, 56864, 57584, 57645, 64004, 65535, 73124, 105524, 107864,
123824, 131144, 164175, 184635, 198315, 214334, 215775, 256274, 286995,
307395, 319275, 347324, 388245, 397485, 407924, 415275, 454124, 491535,
524432, 525986, 546272, 568815, 589407, 679496, 686985, 840255, 914175,
936494, 952575, 983775, 1025504, 1091684, 1231424, 1259642, 1276904, 1390724,
1405845, 1574727, 1659585, 1759874, 1788254, 1925564, 2123583, 2200694,
2388044, 2521694, 2539004, 2619705, 2648204, 2759925, 2792144, 2822715,
2847584, 3104744, 3137355, 3170936, 3240614, 3289934, 3653564, 3693525,
3794834, 3877184, 3988424, 4002405, 4034744, ...
To expand on your Sequence Number 1328, I (n) = I (n+2) : "1, 4, 7, 8,

10, 26, 32, 70, 74, 122, 146, 308, 314, 386, 512, 554, 572, 626, 635, 728,
794, 842, 910, 914, 1015, 1082,"
1226, 1322, 1330, 1346, 1466, 1514, 1608, 1754, 1994, 2132, 2170, 2186,
2306, 2402, 2426, 2474, 2590, 2642, 2695, 2762, 2906, 3242, 3314, 3506,
3746, 3866, 3986, 4034, 4274, 4292, 4338, 4682, 4946, 5114, 5186, 5594,
5714, 5834, 5950, 6122, 6434, 6497, 6506, 6626, 6764, 7034, 7466, 8042,
8114, 8354, 8522, 8546, 8714, 8882, 9100, 9122, 9242, 9758, 9866, 10154,
10202, 10226, 10307, 10466, 10826, 10874, 11162, 11402, 12074, 12146, 12212,
12242, 12266, 12317, 12434, ...
As long as we are on this train of thought, then the next logical sequence
is to include the occurrences of n when o (n+1) = o (n), and it is as follows:
14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364, 14841, 18873,
19358, 20145, 24957, 33998, 36566, 42818, 56564, 64665, 74918, 79826, 79833,
84134, 92685, 109214, 111506, 116937, 122073, 138237, 147454, 161001, 162602,
166934, 174717, 190773, 193893, 201597, 230390, 274533, 289454, 347738,
383594, 416577, 422073, 430137, 438993, 440013, 445874, 455373, 484173,
522621, 544334, 605985, 621027, 649154, 655005, 685995, 695313, 739556,
792855, 937425, 949634, 1154174, 1174305, 1187361, 1207358, 1238965, 1642154,
1670955, 1765664, 1857513, 2168906, 2284814, 2305557, 2913105, 3296864,
3477435, 3571905, 3582224, 3682622, 3726009, 4328937, 4473782, 4481985,
4701537, 4795155, 5002335, 5003738, 5181045, 5351175, 5446425, 5459024,
5517458, 6309387, 6431732, 6444873, 6514995, 6771405, 7192917, 7263944,
7796438, 7845386, 7955492, ...
Continuing is to include the occurrences of n when o (n+2) = o (n), and
it is as follows: 33, 54, 284, 366, 834, 848, 918, 1240, 1504, 2910, 2913,
3304, 4148, 4187, 6110, 6902, 7169, 7912, 9359, 10250, 10540, 12565, 15085,
17272, 17814, 19004, 19688, 21410, 21461, 24881, 25019, 26609, 28124, 30592,
30788, 31484, 38210, 38982, 39786, 40310, 45354, 46863, 49225, 51835, 53106,
53963, 55286, 59987, 76360, 77057, 81055, 83094, 94996, 95392, 96728, 101101,
117570, 117858, 121394, 124758, 127585, 143369, 147340, 149149, 149750,
150419, 163936, 167560, 170114, 170561, 173920, 175796, 181384, 197260,
205727, 215069, 220817, 239954, 278920, 280787, 292315, 293656, 319955,
334540, 334983, 336505, 344416, 359454, 360325, 360685, 370435, 388074,

418307, 434433, 463218, 472323, 477904, 510340, 516026, 543453, 564857, ...

From page 235, the first occurrence of k when n - I (n) = k, and it is

as follows: 3, 4, 9, 6, 25, 10, 15, 12, 21, 0, 35, 18, 33, 26, 39, 24, 65,

34, 51, 38, 45, 30, 95, 36, 69, 0, 63, 52, 161, 42, 87, 48, 93, 0, 75, 54, 217, 74, 99, 76, 185, 82, 123, 60, 117, 66, 215, 72, 141, 0, 235, 0, 329, 78, 159, 98, 105, 0, 371, 84, 177, 122, 135, 96, 305, 90, 427, 134, 201, 102, 335, 108, 213, 146, 207, 148, 245, 114, 511, 152, 189, 130, 395, 164, 165, 0, 415, 120, 581, 126, 267, 132, 261, 138, 623, 144, 1501, 194, 195, 0, 485, ...

The sequence of k's when n - I (n) = k has no solutions (the Zeros above), and it is as follows: 10, 26, 34, 50, 52, 58, 86, 100, 116, 122, 130, 134, 146, 154, 170, 172, 186, 202, 206, 218, 222, 232, 244, 260, 266, 268, 274, 290, 292, 298, 310, 326, 340, 344, 346, 362, 366, 372, 386, 394, 404, 412, 436, 466, 470, 474, 482, 490, 518, 520, 532, 534, 536, 546, 554, 562, 566, 580, 584, 596, 626, 634, 650, 652, 666, 680, 686, 688, 698, 706, 722, 724, 730, 732, 746, 772, 778, 786, 794, 808, 818, 834, 842, 850, 872, 874, 902, 906, 914, 922, 926, 932, 940, 962, 964, 974, 980, 986, 1018, 1036, 1038, ...

The first occurrence of k when o (n) - n = k, and it is as follows:
2, 0, 4, 9, 0, 6, 8, 10, 15, 14, 21, 121, 27, 22, 16, 12, 39, 289, 65, 34, 18, 20, 57, 529, 95, 46, 69, 28, 115, 841, 32, 58, 45, 62, 93, 24, 155, 1369, 217, 44, 63, 30, 50, 82, 123, 52, 129, 2209, 75, 40, 141, 0, 235, 42, 36, 106, 99, 68, 265, 3481, 371, 118, 64, 56, 117, 54, 305, 4489, 427, 134, 201, 5041, 98, 70, 213, 48, 219, 66, 365, 6241, 147, 158, 237, 6889, 395, 166, 105, 0, 171, 78, 581, 88, 267, 116, 445, 0, 245, 9409, 1501, 124, 291, ...

The sequence of k's when o (n) - n = k has no solutions (the Zeros above), and this is also the "untouchable" numbers of Paul Erdos. Wells, page 125. It is as follows: 2, 5, 52, 88, 96, 120, 124, 146, 162, 188, 206, 210, 216, 238, 246, 248, 262, 268, 276, 288, 290, 292, 304, 306, 322, 324, 326, 336, 342, 372, 406, 408, 426, 430, 448, 472, 474, 498, 516, 518, 520, 530, 540, 552, 556, 562, 576, 584, 612, 624, 626, 628, 658, 668, 670, 708, 714, 718, 726, 732, 738, 748, 750, 756, 766, 768, 782, 784, 792, 802, 804, 818, 836, 848, 852, 872, 892, 894, 896, 898, 902, 926, 934, 936, 964, 966, 976, 982, 996, 1002, 1028, 1044, 1046, 1060, 1068, 1074, 1078, 1080, 1102, 1116, 1128, ...

If a number is in parenthesis then that is not the value but the limit to which the test was run on my HP-71B. On the other hand, if the integer presented is Zero, then there is no possible answer. Often, a particular series maybe divided by two to save some room or to make clearer the sequence involved. The references for the above are in your bibliography as SI1, BE3 and AS1, plus "The Penguin Dictionary of Curious and Interesting Numbers," David Wells, Middlesex, England, 1986. If at some future date, I run across a filler or greatly extend the limits, I will forward the same to you.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:hp110+

Quotes:

"God invented 1,2 and 3, and man invented all the rest."
The author is unknown to me, but is a great quote for a book on numerical sequences. Maybe I'm thinking of the following quote.

"God himself made the whole numbers: everything else is the work of

man." Leoplod Kronecker
"The trouble with integers is that we have examined only the small ones." Ronald Graham
"The primary source of all mathematics are the integers." Herman Minkowski
"I am ill at these numbers." Wm. Shakespeare (Hamlet)

1 January 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Sir, this is a updated version of an earlier letter.
In the above referenced text, three sequences are cited (correctly, I might add) as simply "A Self-Generating Sequence." They are SSN (for Sloane Sequence Nbr. and not for Social Security Nbr.) 201, 231, and 909. In "Computers in Number Theory" edited by A.O.L. Atkin and B.J. Birch, Academic Press 1971, pages 249 to 257, (your reference is AT1) these three sequences are called "Ulam's Summation Sequences." U2({1,2}) is SSN 201, U2({2,3}) is SSN 231, and U2({1,3}) is SSN 909. However on page 256 a fourth sequence U3({1,2,3}) is mentioned but not cited by you. Therefore; please consider the following sequences for your upcoming second edition: U3({1,2,3}), U3({1,2,4}), U3({1,3,4}), U3({2,3,4}), U4({1,2,3,4}), U5({1,2,3,4,5}), U6({1,2,3,4,5,6}), U7({1,2,3,4,5,6,7}), U8({1,2,3,4,5,6,7,8}), U9({1,2,3,4,5,6,7,8,9}), U10({1,2,3,4,5,6,7 U11({1,2,3,4,5,6,7,8,9,10,11}) and U12({1,2,3,4,5,6,7,8,9,10,11,12})..

U3({1,2,3}): 1, 2, 3, 6, 9, 10, 11, 12, 28, 29, 30, 53, 56, 57, 80, 82, 104, 105, 107, 129, 130, 132, 154, 155, 157, 179, 180, 182, 204, 205, 207, 229, 230, 232, 254, 255, 257, 279, 280, 282, 304, ... (the nth term for n>5: 3*n= 25*n -45, 3*n+1= 25*n -43, and 3*n+2= 25*n -21, Muller's Theorem) ... .

U3({1,2,4}): 1, 2, 4, 7, 10, 12, 16, 17, 32, 36, 42, 57, 72, 73, 98, 102, 104, 129, 159, 164, 174, 189, 199, 221, 224, 255, 286, 287, 347, 372, 378, 403, 428, 443, 444, 469, 494, 529, 560, 586, 592, 652, 683, 718, 743, 780, 784, 865, 871, 957, 963, 988, 1085, 1110, 1114, 1155, 1176, 1206, 1236, 1293, 1302, 1460, 1516, 1541, 1633, 1639, 1791, 1827, 1852, 1882, 1918, 1978, 2009, 2035, 2070, 2126, ...

U3({1,3,4}): 1, 3, 4, 8, 12, 13, 15, 16, 18, 40, 42, 52, 77, 78, 79, 87,114, 116, 117, 152, 154, 188, 224, 228, 230, 260, 262, 263, 300, 301, 302, 336, 338, 375, 407, 409, 411, 412, 444, 446, 481, 482, 517, 521, 554, 555, 591, 626, 630, 631, 663, 665, 700, 701, 736, 740, 773, 774, 810, 845, 849, 850, 882, 884, 919, 920, 955, 959, 992, 993, 1029, ...(the nth term for n>8: 4*n+1=73*n-249 and if n=0 mod 3 then +3, 4*n+2= the previous term +1, 4*n+3= 73*n -213 and if n=2 mod 3 then +1, and 4*n+4= 73*n-177 and if n=0 mod 3 then +34 or if n=1 mod 3 then +32) ...

U3({2,3,4}): 2, 3, 4, 9, 14, 15, 16, 19, 23, 24, 55, 59, 60, 63, 64, 104, 105, 109, 112, 114, 155, 157, 159, 160, 203, 204, 206, 207, 208, 253, 254, 255, 258, 302, 305, 307, 308, 351, 352, 354, 355, 356, 401, 402, 403, 406, 450, 453, 455, 456, 499, 500, 502, 503, 504, 549, 550, 551, 554, 598, 601, 603, 604, 647, 648, 650, 651, 652, 697, 698, 699, 702, 746, 749, 751, 752, 795, 796, 798, ... (the nth term for n>1: 13*n= 148*n -92, 13*n+1= 148*n -90, 13*n+2= 148*n -89, 13*n+3=148*n -88, 13*n+4= 148*n -43, 13*n+5= 148*n -42, 13*n+6= 148*n -41, 13*n+7= 148*n -38, 13*n+8= 148*n +6, 13*n+9= 148*n +9, 13*n+10= 148*n +11, 13*n+11= 148*n +12, and 13*n+12= 148*n +55) ...

U4({1,2,3,4}): 1, 2, 3, 4, 10, 16, 17, 18, 19, 22, 64, 65, 66, 68, 69, 128, 132, 188, 190, 191, 194, 252, 253, 255, 313, 314, 318, 376, 377, 380, 438, 439, 498, 554, 556, 557, 560, 561, 620, 621, 680, 684, 740, 742, 743, 745, 803, 804, 863, 869, 923, 924, 925, 927, 928, 929, 988, 990, 1048, 1049, ...

U5({1,2,3,4,5}): 1, 2, 3, 4, 5, 15, 25, 26, 27, 28, 29, 35, 43, 45, 165, 171, 172, 174, 180, 181, 328, 333, 338, 339, 340, 341, 493, 499, 500, 647, 652, 657, 658, 659, 660, 661, 662, 663, 815, 818, 819, 971, 1127, (and no others less than 1130) ... .

U6({1,2,3,4,5,6}): 1, 2, 3, 4, 5, 6, 21, 36, 37, 38, 39, 40, 41, 51, 61, 66, 284, 285, 289, 290, 297, 298, 299, 310, 312, 559, 561, 562, 570, 571, 574, 575, 834, 835, 837, 838, 839, 840, 841, 849, 850, (and no others less than 1095) ... .

U7({1,2,3,4,5,6,7}): 1, 2, 3, 4, 5, 6, 7, 28, 49, 50, 51, 52, 53, 54, 55, 70, 82, 91, 109, 112, 555, 556, 563, 564, 572, 573, 576, 583, 584, 591, 593, (and no others less than 1053) ... .

U8({1,2,3,4,5,6,7,8}): 1, 2, 3, 4, 5, 6, 7, 8, 36, 64, 65, 66, 67, 68, 69, 70, 71, 92, 106, 120, 141, 148, 792, 793, 802, 803, 816, 817, (and no others less than 818) ... .

U9({1,2,3,4,5,6,7,8,9}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 45, 81, 82, 83, 84, 85, 86, 87, 88, 89, 117, 133, 153, 177, 189, 221, 225, (and no others less than 1139), ...

U10({1,2,3,4,5,6,7,8,9,10}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 55, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 145, 163, 190, 217, 235, 271, (and no others less than 278) ... .

U11({1,2,3,4,5,6,7,8,9,10,11}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 66, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 176, 196, 231, 261, 286, 326, 341, 391, 396, (and no others less than 1849) ... .

U12({1,2,3,4,5,6,7,8,9,10,11,12}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 78, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 210, 232, 276, 309, 342, 386, 408, 463, 474, (and no others less than 2473) ... .

for n>2, Un({1,2, ... ,n-1,n}): 1, 2, ... , n-1, n, °n(n+1), n^2 to n^2+n-1, (3*n^2-n)/2, (3*n^2+3*n-4)/2 for n>4, 2*n^2-n for n>4, 2*n^2+2*n-3 for n>6, (supposition only) ... .

These are all infinite series, provable along the lines employed by Euclid (Elements, Proposition 20, Book IX) to demonstrate the infinitude of the primes. Assume that X is the largest number in the sequence belonging to the Summation Series of n numbers at a time. Since these Xs are in numerical order, then X + X + ... + X + X represents a uniquely describable number. Therefore the assumption that X is that largest number in the sequence is false.

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF & GI
RGWv:hp110+
Rev:01-Supr22Sept92

31 August 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
In "Computers and the Imagination," Clifford A. Pickover, St. Martin's Press, Inc., New York, 1991, on pages 352-4, there is a "Grasshopper sequence" which grows from a binary tree. The sequence is as follows:
1, 4, 10, 12, 22, 26, 30, 46, 54, 62, 66, 78, 94, 110, 126, 134, 138, 158, 162, 186, 190, 222, 254, 270, 278, 282, 318, 326, 330, 374, 378, 382, 402, 446, 474, 510, 542, 558, 566, 570, 638, 654, 662, 666, 750, 758, 762, 766, 806, 810, 834, 894, 950, 954, 978, 1022, 1086, 1118, 1122, 1134, 1142, 1146, 1278, 1310, 1326, 1334, 1338, 1502, 1518, 1526, 1530, 1534, 1614, 1622, 1626, 1670, 1674, 1698, 1790, 1902, 1910, 1914, 1958, 1962, 1986, 2046, 2174, 2238, 2246, 2250, 2270, 2274, 2286, 2294, 2298, 2418, 2558, 2622, 2654, 2670, 2678, 2682, 2850, 3006, 3038, 3054, 3062, 3066, 3070, 3230, 3246, 3254, 3258, 3342, 3350, 3354, 3398, 3402, 3426, 3582, 3806, 3822, 3830, 3834, 3918, 3926, 3930, 3974, 3978, 4002, 4094, 4350, 4478, 4494, 4502, 4506, 4542, 4550, 4554, 4574, 4578, 4590, 4598, 4602, 4838, 4842, 4866, 5010, 5118, 5246, 5310, 5342, 5358, 5366, 5370, 5702, 5706, 5730, 5874, 6014, 6078, 6110, 6126, 6134, 6138, 6142, 6462, 6494, 6510, 6518, 6522, 6686, 6702, 6710, 6714, 6738, 6798, 6806, 6810, 6854, 6858, 6882, 7166, 7614, 7646, 7662, 7670, 7674, 7838, 7854, 7862, 7866, 7950, 7958, 7962, 8006, 8010, 8034, 8190, 8702, 8958, 8990, 9006, 9014, 9018, 9086, 9102, 9110, 9114, 9150, 9158, 9162, 9182, 9186, 9198, 9206, 9210, 9678, 9686, 9690, 9734, 9738, 9762, 10022 (the 225th term), ...
Sequentially yours,
Robert G. Wilson v,
Ph. D., ATP/CF & GI
RGWv:hp110+

22 December 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forth-coming second edition of the above. "A positive integer is called 'Powerful' if and only if it can be expressed as a sum of positive integral powers of its digits." A favorite example of a dear friend of mine is 153 = 1^3 + 5^3 + 3^3. "Clearly, the positive integers from one to nine are trivially powerful, since each can be written as its own first power."

The sequence is as follows: (1, 2, 3, 4, 5, 6, 7, 8, 9) 24, 43, 63, 89, 132, 135, 153, 175, 209, 224, 226, 262, 264, 267, 283, 332, 333, 334, 357, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 407, 445, 463, 518, 598, 629, 739, 794, 849, 935, 994, 1034, 1073, 1074, 1234, 1255, 1306, 1323, 1326, 1349, ... For a further rendition, consult the attached pages for the first 2327 terms which are all the "Powerful" numbers less than 60,000.

Some of the above are represented by two or more uniquely different set of exponents, some are primes, some are sequential for a time, some are represented by exponents equal to the some combination of digits of the integer, some are represented by contiguous exponents, and some are repre-sented by exponents equal to the number of digits of the integer. Each of these are subsets of the above and represent all of the members that are less than 100,000.

The first of these six sequences (two or more uniquely different set of exponents) is as follows: 264, 373, 375, 935,

2139, 2203, 2223, 2243, 2245, 2263, 2332, 2334, 2336, 2352, 2356, 2372, 2376, 2394, 2407, 2427, 2536, 2537, 2664, 2733, 2793, 2824, 2843, 2932,

2933, 2939, 3257, 3292, 3298, 3324, 3342, 3435, 3437, 3457, 3477, 4132, 4224, 4225, 4228, 4243, 4245, 4249, 4262, 4265, 4288, 4332, 4352, 4353, 4355, 4397, 4572, 4573, 4732, 4733, 4933, 4959, 5323, 5325, 5432, 5832, 6424, 6793, 6934, 6939, 7329, 7639, 7906, 8208, 8224, 8225, 8262, 8283, 8289, 8376, 8396, 8825, 9273, 9274, 9324, 9342, 9723,

12324, 12347, 13933, 14363, 16293, 16363, 16432, 16434, 16472, 16479, 16524, 16743, 16927, 17243, 17247, 17248, 17249, 17848, 19243, 19732, 19733,

20933, 21258, 22243, 22247, 22249, 22283, 22285, 22338, 22373, 22375, 22395, 22465, 22489, 22539, 22593, 22628, 22732, 22733, 22738, 22848, 22864, 22883, 22935, 22953, 22954, 22955, 22958, 22995, 23235, 23237, 23253, 23275, 23295, 23297, 23345, 23387, 23417, 23433, 23437, 23472, 23473, 23474, 23476, 23492, 23493, 23497, 23498, 23543, 23548, 23562, 23582, 23584, 23634, 23652, 23697, 23723, 23749, 23835, 23852, 23856, 23875, 23897, 23924, 23928, 23943, 23945, 23947, 23949, 23967, 23988, 24137, 24226, 24229, 24265, 24289, 24375, 24376, 24379, 24396, 24397, 24483, 24489, 24532, 24536, 24593, 24622, 24623, 24627, 24628, 24644, 24647, 24664, 24667, 24680-9, 24736, 24756, 24759, 24796, 24823, 24843, 24845, 24848, 24864, 24865, 24868, 24883, 24953, 24976,

25235, 25254, 25257, 25273, 25432, 25437, 25439, 25453, 25473, 25476, 25479, 26224, 26243, 26246, 26247, 26339, 26352, 26393, 26739, 26537, 26593, 26793, 26713, 27234, 27322, 27346, 27363, 27367, 27369, 27473, 27568, 27923, 27984, 28264, 28265, 28395, 28425, 28533, 29637, 29967,

30349, 32235, 32252, 32275, 32297, 32436, 32524, 32749, 32786, 32812, 32813, 32814, 32816, 32830-9, 32852, 32854, 32857, 32859, 32872, 32873, 32874, 32877, 32890-9, 32907, 32908, 32922, 32924, 32926, 32942, 32943, 32945,

32948, 32949, 32968, 32969, 32985, 32987, 32989, 33022, 33028, 33042, 33225, 33227, 33228, 33229, 33246, 33262, 33263, 33266, 33268, 33284, 33398, 33427, 33428, 33429, 33442, 33486, 33539, 33592, 33752, 33798, 33804, 33822, 33828, 33829, 33842, 33844, 33847, 33882, 34234, 34236, 34238, 34239, 34256, 34272, 34292, 34297, 34298, 34299, 34222, 34326, 34474, 34525, 34727, 34728, 34746, 34762, 34872, 34892, 34922, 34924, 34982, 35223, 35224, 35225, 35245, 35247, 35248, 35284, 35287, 35352, 35625, 35628, 35932, 35938, 35939, 35992, 36292, 36324, 36342, 36382, 36385, 36472, 36722, 36723, 36724, 36922, 36984, 37223, 37224, 37226, 37228, 37229, 37242, 37243, 37244, 37248, 37264, 37284, 37352, 37372, 37424, 37426, 37428, 37442, 37482, 37648, 38322, 38522, 39338, 39352, 39372, 39392, 39393, 39402, 39422, 39423, 39427, 39428, 39429, 39442, 39446, 39462, 39482, 39483, 39624, 39628, 39642, 39662, 39538, 39682, 39752, 39758, 39823, 39844, 39882, 39886,
40629, 40689, 42243, 42248, 42263, 42264, 42268, 42539, 42624, 42689, 42759, 43498, 43836, 43926, 44648, 46712, 46716, 46730-9, 46753, 46792, 46794, 46796, 46824, 46825, 46932, 46933, 46935, 46953, 46973, 46975, 46976, 47236, 47363, 47653, 47693, 47726, 48625, 49323, 49436, 49637, 49722, 49725, 49728, 49876, ...

The sequence with three uniquely different set of exponents is as follows:
2203, 2332, 4224, 4228, 4245, 8224, 13933, 16432, 19733, 22465, 22489, 22953, 22954, 23235, 23237, 23275, 23437, 23472, 23473, 23492, 23584, 23835, 23852, 23924, 23943, 24265, 24289, 24376, 24623, 24647, 24683, 24684, 24687, 24843, 25254, 25479, 26243, 26339, 26793, 27363, 28265, 28425, 32813, 32830, 32831, 32832, 32835, 32838, 32857, 32873, 32877, 32894, 32949, 32987, 33262, 33268, 33284, 33592, 33752, 33842, 34326, 34872, 35932, 37223, 37228, 37442, 38322, 39483, 39538, 42243, 42624, 46732, 46733, 46825, 46935, 46973, 49728, ...

The sequence with four uniquely different set of exponents is as follows:
2334, 2537, 4243, 22243, 22247, 22249, 23493, 23497, 23875, 23897, 23943, 24229, 24628, 24736, 24756, 24845, 24848, 26247, 28264, 32816, 32833, 32834, 32836, 32943, 33227, 34239, 36324, 36724, 37226, 37242, 37428, 39429, 39442, 39844, 46932, ...

The sequence with five uniquely different set of exponents is as follows:
25432, 26224, 32524, 32812, 32874, 32922, 32924, 32948, 32985, 33829, 39423, 39428, 39642, 42264, ...

The sequence with six uniquely different set of exponents is as follows:
22864, 24823, 24883, 32837, 32892, 32893, 32896, 39482, ...

The sequence with seven uniquely different set of exponents is as follows:
23852, 35224, ...

The sequence that follows are those members occurring first of the original set (see the first term of the previous seven sequences) which can be represented by n uniquely different set of exponents: 24, 264, 2203, 2334, 25432, 22864, 23852, ...

The sequence that follows are those members occurring first of the full decade set (see only ten "Powerful" numbers on a subsequent page) which can be represented by n uniquely different set of exponents: 370, 24680, 32830, ...

The second of these six sequences (the primes) is as follows:
43, 89, 283, 373, 379, 463, 739, 2063, 2083, 2131, 2137, 2179, 2203, 2243, 2333, 2423, 2753, 2803, 2843, 2939, 2953, 3167, 3257, 3457, 3527, 3943, 3947, 4133, 4153, 4157, 4159, 4241, 4243, 4261, 4339, 4373, 4397, 4447, 4649, 4733, 4933, 4951, 4957, 5323, 5653, 5657, 6247, 6793, 7639, 8209, 8243, 8263, 8443, 8623, 8647, 8971, 9697,
10723, 12343, 12347, 13337, 13933, 14369, 16363, 16433, 16453, 16477, 16729, 16927, 17443, 17483, 17737, 18617, 20483, 20887, 21323, 22247, 22283, 22447, 22573, 22643, 22973, 23053, 23297, 23327, 23417, 23473, 23497, 23549, 23563,

23743, 23747, 23857, 24043, 24137, 24229, 24337, 24373, 24379, 24593, 24623, 24683, 24733, 24841, 24847, 24953, 24977, 25127, 25237, 25253, 25367, 25439, 25453, 25657, 26153, 26249, 26339, 26393, 26647, 26713, 27253, 27361, 27367, 27479, 27527, 27617, 27697, 27749, 28403, 28513, 28603, 29347, 29947, 30341, 30347, 32297, 32327, 32587, 32783, 32789, 32831, 32833, 32839, 32941, 32969, 32983, 32987, 33023, 33029, 33223, 33247, 33427, 33623, 33757, 33829, 34217, 34231, 34297, 34327, 34367, 34439, 34981, 35081, 35083, 35089, 35221, 35227, 35447, 35809, 35933, 36277, 36343, 36457, 36473, 36497, 36637, 36877, 37223, 37243, 37423, 37483, 37663, 37997, 38567, 39373, 39443, 39623, 39827, 39869, 42407, 42683, 42689, 42863, 42953, 44449, 44641, 44647, 44683, 46771, 46933, 46993, 47363, 47639, 47653, 47699, 49727, 49891, 52583, 52639, 52709, 52783, 52837, 54727, 55843, 56633, 59063, 59069, 59243, 59263, 59333, ...

The third of these six sequences are a finite number of consecutive (siblings?) integers.

The sequence, which I will term "twins" after the example of the primes, consists with only two "Powerful" numbers in a row, is as follows: 1073, 2263, 2314, 2352, 2355, 2372, 2464, 2536, 2863, 3214, 3254, 3294, 3453, 3725, 3764, 4132, 4224, 4352, 4396, 4648, 4732, 4735, 4932, 5258, 5366, 5832, 6423, 6934, 8262, 8288, 9236, 10693, 10722, 13132, 14362, 15422, 16742, 17463, 19732, 20355, 20932, 21235, 21348, 22337, 22557, 22732, 22972, 22994, 23274, 23347, 23364, 23476, 23492, 23522, 23548, 23562, 23763, 23852, 23924, 24264, 24336, 24352, 24372, 24375, 24396, 24643, 24936, 24976, 25253, 25272, 25275, 25322, 25366, 25476, 26393, 26426, 27472, 27497, 28532, 28553, 29346, 29435, 32436, 32524, 32527, 32587, 32907, 32926, 32968, 33246, 33283, 33538, 33685, 33778, 33828, 33846, 33864, 34322, 34326, 34366, 34387, 34525, 34528, 34727, 34947, 35247, 35284, 35487, 35978, 36253, 36274, 36342, 36437, 36567, 36656, 36742, 37267, 37352, 37392, 37442, 37826, 38348, 38382, 38566, 38834, 39372, 39392, 39442, 39532, 39623, 42598, 42623, 42863, 43473, 43527, 46372, 46682, 46862, 46993, 47362, 47367, 47652, 47967, 49385, 49784, 52522, 53262, 53862, 54328, 54632, 56858, 57262, 59138, 59192, 59202, 59226, 59282, 59332, 59335, 59356, 59442, 59462, ...

The sequence, which I will term "triples" in keeping with the previous, with only three "Powerful" numbers in a row is as follows: 332, 2062, 2223, 2394, 2626, 2737, 2792, 2932, 3435, 4572, 4622, 5342, 6792, 8223, 9272, 23235, 23472, 23496, 23582, 23743, 23747, 23856, 24735, 24863, 25235, 27322, 27345, 32325, 32872, 32922, 33427, 33622, 33822, 33842, 34872, 36472, 36722, 37242, 38324, 39627, 46823, 49236, 49322, 52832, 56632, 59222, ...

The "quadruples" sequence with only four "Powerful" numbers in a row is as follows: 3474, 22332, 23324, 37262, 39482, 52723, ...

The "quituples" sequence with only five "Powerful" numbers in a row is as follows: 2332, 3234, 22372, 22535, 23435, 37482, ...

The "dectuples" sequence with only ten "Powerful" numbers, a full decade (in a row having as its first member a number congruent to zero modulus ten) is as follows: 370, 2130, 2240, 3940, 4150, 4240, 4260, 4950, 5320, 8200, 8970, 16430, 16470, 17240, 17840, 22950, 24590, 24620, 24680, 24750, 24840, 25430, 26240, 27360, 28260, 28730, 30340, 32780, 32810, 32830, 32890, 32940, 32980, 33020, 33220, 33260, 34230, 34290, 34980, 35080, 35220, 35930, 36980, 37220, 37420, 39420, 42240, 42260, 44640, 46710, 46730, 46770, 46790, 46930, 46970, 47690, 49720, 49890, 52700, 53480, 59060, 59240, 59790, 59930, 60350, 62640, 62660, 63290, 63320, 63470, 63760, 64730, 66350, 66480, 68640, 75990, 78540, 78650, 79590, 82240, 82370, 82440, 83340, 84950, 85230, 91820, 93420, 94520, 95240, 98340, 98890, ...

And that's it for the "tuples!"

This sequence consists of decades (plus the number of terms - cousins?) which have more than two integers in it but less then ten integers and not included singularly in any of the previous set of five sequences immediately

proceed-ing this one: 263, 2063, 2224, 2354, 2374, 2443, 2533, 2573, 2624, 2735, 2934, 3254, 3294, 3434, 3453, 3523, 3723, 4133, 4223, 4443, 4643, 4735, 5363, 6243, 6935, 6973, 8224, 8283, 9233,
12343, 14363, 17333, 17373, 17443, 19733, 20353, 20533, 21343, 22243, 22263, 22336, 22376, 22463, 22536, 22593, 22734, 22973, 23235, 23273, 23293, 23344, 23436, 23475, 23495, 23524, 23543, 23563, 23584, 23746, 23855, 23923, 23944, 24334, 24375, 24483, 24643, 24663, 24735, 24864, 24933, 25234, 25253, 25274, 25323, 25474, 25653, 26333, 27324, 27345, 27473, 27493, 28533, 29344, (<30000) ... .

The group of series that follow measure the distance or "gap" between neighbors.

This sequence consists of two numbers seperated by one: 224, 262, 2203, 2225, 2353, 2443, 2572, 2733, 2735, 3145, 3165, 3212, 3252, 3255, 3292, 3523, 3525, 3723, 4353, 4403, 4447, 4733, 4736, 5364, 6247, 6932, 6935, 6937, 6972, 6974, 7962, 8243, 9234, 9723, 12343, 12345, 12432, 17332, 17376,
17443, 22247, 22267, 22283, 22335, 22376, 22393, 22463, 22465, 22533, 22593, 22595, 22733, 22753, 22823, 23034, 23162, 23237, 23272, 23295, 23343, 23345, 23433, 23452, 23474, 23527, 23745, 23835, 23943, 23945, 23947, 24332, 24334, 24373, 24487, 24667, 24733, 24737, 24994, 25273, 25477, 25657, 26337, 26352, 27324, 27343, 27347, 27747, 28423, 29347, 32525, 32635, 32852, 32857, 32903, 32905, 32924, 32963, 33284, 33286, 33824, 33826, 33844, 33882, 34036, 34254, 34272, 34472, 34526, 34672, 34784, 34832, 34834, 34922, 35022, 35203, 35243, 35245, 35285, 35287, 35885, 35992, 36232, 36237, 36275, 36292, 36294, 36346, 36435, 36746, 36922, 37244, 37246, 37265, 38322, 38343, 39352, 39536, 39687, 39823, 39825, 39882, 39884, 42534, 42687, 42864, 43236, 43432, 43523, 43525, 43542, 43782, 43924, 47944, 49234, 49272, 49274, 52362, 52743, 53222, 53224, 53226, 53647, 54326, 59224, 59267, 59312, 59333, 59373, 59392, ...

This sequence consists of two numbers seperated by two: 132, 264, 1323, 2373, 2376, 2574, 2623, 3295, 3342, 3432, 3454, 3566, 3746, 4133, 4225, 4285, 6244, 7296, 8225, 17445, 19733, 20352, 20536, 21345, 21436, 22355,
22735, 22932, 23053, 23232, 23292, 23493, 23584, 23634, 23853, 23925, 24226, 24376, 24423, 24464, 24644, 24664, 24773, 24865, 25232, 25254, 25473, 26736, 27234, 27253, 27433, 28533, 29343, 29634, 32562, 32854, 32874, 32965, 33084, 33772, 33775, 33954, 34323, 34674, 35205, 35625, 35845, 36234, 36324, 36343, 36382, 36454, 36544, 36743, 37642, 37645, 38345, 38563, 39443, 39533, 39624, 42536, 42595, 48664, 49295, 52523, 52745, 52834, 53246, 53263, 56273, 56294, 59283, 59336, 59353, 59423, ...

This sequence consists of two numbers seperated by three: 1672, 2423, 2445, 2532, 2753, 3492, 3674, 4443, 4644, 5232, 5653, 5873, 9474, 9693, 14722, 17334, 17372, 20532, 22243, 22263, 22624, 23523, 23723, 23963, 24483, 24532, 24572, 24712, 24792, 24932, 25323, 25342, 25653, 26333, 26533, 27493,
27635, 28332, 32252, 32899, 33242, 34762, 34924, 36254, 36474, 37332, 39844, 40685, 42624, 42683, 43232, 43434, 43474, 43582, 43674, 46805, 47363, 49364, 49872, 49922, 52364, 54655, 56382, 59263, 59463, 59602, ...

This sequence consists of two numbers seperated by four: 2934, 3652, 5562, 8283, 9342, 13332, 17352, 20372, 22973, 23543, 23563, 23652, 23892, 25384, 25672, 25764, 26443, 33422, 33442, 33752, 34274, 36412, 36492, 37353, 38522, 39682, 39864, 40624, 47962, 52763, 56634, ...

This sequence consists of two numbers seperated by five: 2843, 4397, 7323, 14363, 23673, 27473, 27562, 29253, 32743, 33532, 33792, 33798, 34562, 35352, 35372, 37443, 37732, 39642, 39752, 43492, 46799, 46953, 47673, 52783, 54363, 54742, 56433, ...

This sequence consists of two numbers seperated by six: 4332, 7632, 24796, 39462, 52432, 52693, ...

This sequence consists of two numbers seperated by seven: 24599, 28395, ...

This sequence consists of two numbers seperated by eight: 2194, 2794, 39393,

42599, 59193, ...
This sequence consists of two numbers seperated by nine: 2597, 23897, 28593, 47296, 52294, 53396, ...
This sequence consists of two numbers seperated by ten: 2396, 4249, 24669, 24739, 27349, 32819, 32969, 34279, 42249, 46719, 46779, 46959, 47679, 53469, ...
Here is the series which represents the "gap" (see the first term in each of the above) of increasing magnitude:
332, 224, 132, 1672, 2934, 2843, 4332, 24599, 2194, 2597, 2396, 4228, 357, 2048, 209, 267, 1306, 135, 24, 43, 1234, 153, 334, 6579, 4574, 63, 3496, 379, 17378, 16494, 598, 25494, 6939, 175, 9237, 226, 19736, 407, 1034, 994, 25389, 16942, 89, 1498, 3167, 6603, 3385, 5384, 283, 5182, 1255, 22395, 12562, 26593, 463, 6976, 43926, 21439, 935, 6732, 12434, 49436, 4869, 26249, 5567, 5766, 16363, 2864, 2664, 3786, 1765, 20572, 38383, 8462, 20752, 42407, 27052, 62976, xxxxx, 518, xxxxx, xxxxx, 6849, 3856, xxxxx, 849, 4485, 4738, 1676, 54476, 7032, 1542, xxxxx, 5453, 16634, xxxxx, xxxxx, 58962, xxxxx, xxxxx, 25942, 43983, 17737, 30236, 27129, 20377, 17026, 20644, 56749, 629, 7123, 1386, 28132, 16743, 57263, ...
The fourth sequence (not an infinite one, I'm sorry) is represented by exponents the same as some combination of digits of the integer: 1364, 3435, 4155, 4316, 17463, (<35000), ... , 438579088, ...
The fifth sequence is represented by exponents (in order are under-lined) which are contiguous (rearrengement allowed): 24, 43, 63, 89, 135, 175, 267, 332, 357, 518, 598, 849, 1034, 1073, 1074, 1234, 1306, 1326, 1364, 1672, 1676, 2427, 2537, 2574, 2577, 3295, 3454, 3457, 4403, 5366, 6424, 6443, 6714, 6793, 6972, 7944, 7964, 10693, 10694, 14369, 14697, 14786, 16363, 17026, 17157, 17407, 18436, 18617, 18697, 18946, 21417, 30340, 30341, 32524, 32873, 33068, 34498, 34836, 35932, 36980, 36981, 41382, 43498, 47016, 48464, ...
Numbers whose exponents are just (different) even numbers: 1255, 1306, 1634, 2355, 3706, 4114, 4194, 4228, 4288, 4339, 4449, 4469, 4644, 6649, 6669, 6934, 7906, 8208, 8224, 8289, 9474, 10694, 13133, 13173, 13933, 19733, 20355, 21235, 21835, 22195, 22315, 22335, 22338, 22395, 22465, 22539, 22864, 22935, 22954, 22978, 23215, 23275, 23232, 23235, 23295, 23562, 23835, 23875, 24229, 24289, 24628, 24848, 24868, 25323, 29835, 32235, 32859, 32922, 46770, 46771, 48625, ...
Numbers whose exponents are different (in order) odd numbers: 132, 375, 463, 518, 1542, 2048, 2062, 2083, 2114, 2203, 2205, 2240-9, 2315, 2423, 2427, 2733, 2738, 2803, 2953, 3292, 3940-9, 5182, 5320-9, 5343, 5384, 5832, 5902, 8200-9, 8712, 10693, 10723, 12432, 13532, 17026, 17157, 17240-9, 17287, 17352, 17424, 17443, 20843, 21348, 21439, 21564, 22378, 22593, 22953, 24176, 24622, 24680, 24681, 24953, 25127, 25214, 25257, 25327, 26426, 27052, 28423, 32235, 32326, 33022, 33042, 36324, 36343, 37226, 38124, 38762, 42623, 42662, 49890, 49891, ...
This sequence are those numbers whose exponents are all the same: 153, 370, 371, 407, 1634, 4150, 4151, 8208, 9474, ... , 54748, 92727, 93084, ... , 194979, ...
Near misses, that is they have exponents that are all the same expect for one: (24, 43, 63, 89) 135, 175, 209, 226, 262, 264, 283, 334, 372-9, 445, 518, 629, 739, 794, 935, 1034, 1073, 1074, 1255, 1306, 1386, 1498, 1634, 1765, 1836, 2064, 2114, 2130, 2131, 2139, 2179, 2203, 2355, 2407, 2517, 2536, 2607, 2932, 3525, 3543, 3612, 3706, 3786, 4114, 4152-9, 4194, 4245, 4405, 4572, 4735, 4950, 4951, 6579, 6649, 6669, 6934, 7906, 8200-9, 8224, 8289, 8316, 8756, 8825, 8864, 8970, 8971, 9272, 9947, 13132, 16430, 16431, 16470, 16471, 17133, 17840, 17841, 17914, 22315, 22243, 23587, 23812, 24848, 25764, 27549, 32780-9, 32907, 36385, 37002, 37732, 38456, 40608, 41987,

46770, 46771, 51862, 52700, 52701, 59516, 59606, 59930, 59931, ...

Like any large set of numbers (I'm thinking of the primes here) there are a large, all be it finite, number of subsets. Such is the case here. Unlike the primes, the "powerfuls" are interesting and curious, but are not the fundemental building blocks that the primes represent. Therefore, I shall stop with this next one.

Finally, this sixth sequence (exponents equal to the number of digits/ places of the integer), also know as "Armstrong" or "Pluperfect" numbers, (again, not an infinite one, I'm sorry) is as follows: 153, 370, 371, 407, 1634, 8208, 9474, 54748, 92727, 93084, 548834, 1741725, 4210818, 9800817, 9926315, 24678050, 24678051, 88593477, 146511208, 472335975, 534494836, 912985153, 4679307774, 32164049650, 32164049651, 40028394225, 42678290603, 44708635679, 49388550606, 82693916578, 94204591914, 28116440335967, 4338281769391370, 4338281769391371, 21897142587612075, 35641594208964132, 35875699062250035, 1517841543307505039, 3289582984443187032, 4498128791164624869, 4929273885928088826, 63105425988599693916, 128468643043731391252, 449177399146038697307, 2188769684112291628, 27879694893054074471405, 27907865009977052567814, 28361281321319229463398, 35452590104031691935943, and no others less that 2^80, ...

Reference: Journal of Recreational Mathematics, v10n3p209 by Steven Kahan, Flushing, NY, 1977.

Journal of Recreational Mathematics, v14n1p4-10 by David E. Kullman, Oxford, Ohio, 1981.

"Finding Pluperfect Digital Invariants: Techniques, Results, and Observations," Lionel E. Deimel, Jr., and Michael T. Jones, Journal of Recreational Mathematics, v14n2p87-107, 1982.

Dictionary of Curious and Interesting Numbers, David Wells, Penguin, 1986.

Jean-Pierre Lamoitier, Fifty BASIC Exercises, SYBEX, 1981.

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

11 September 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following sequence for inclusion in your forthcoming second edition of the above. Beginning op page 231, Mr. Pickover presents the "Juggler" problem.
The sequence is as follows: 1, 6, 2, 5, 2, 4, 2, 7, 7, 4, 7, 4, 7, 6, 3, 4, 3, 9, 3, 9, 3, 9, 3, 11, 6, 6, 6, 9, 6, 6, 6, 8, 6, 8, 3, 18, 3, 14, 3, 5, 3, 6, 3, 6, 3, 6, 3, 11, 5, 11, 5, 11, 5, 11, 5, 5, 5, 11, 5, 11, 5, 5, 3, 5, 3, 11, 3, 14, 3, 5, 3, 8, 3, 8, 3, 19, 3, 8, 3, 10, 8, 8, 8, 11, 8, 10, 8, 11, 8, 11, 8, 11, 8, 8, 8, 11, 8, 11, 8, 8, 8, ...
The alternate sequence is as follows: 1, 6, 2, 5, 2, 13, 7, 10, 7, 4, 7, 6, 3, 9, 3, 9, 3, 12, 3, 9, 6, 9, 6, 19, 6, 9, 6, 9, 6, 16, 3, 5, 3, 8, 3, 16, 3, 5, 3, 14, 3, 11, 14, 11, 14, 5, 14, 14, 14, 14, 14, 5, 14, 5, 14, 11, 8, 11, 8, 8, 8, 8, 8, 11, 8, 11, 8, 8, 8, 8, 8, 21, 11, 21, 11, 8, 11, 8, 11, 19, 11, 11, 11, 8, 11, 11, 11, 11, 11, 11, 8, 19, 8, 8, 8, 10, 8, 10, 8, ...
Reference: Clifford A. Pickover, Computers and the Imagination, St. Martin's Press, Inc., NY., 1991
Sequentially yours,
Robert G. Wilson v,
Ph. D., ATP/CF & GI
RGWv:hp110+

23 September 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
In the above referenced text, the most natural sequence is cited as simply "The Natural Numbers." It is SSN (for Sloane Sequence Nbr. and not for Social Security Nbr.) 173. There are fifteen elementary additional sequences that are all very simple. In fact, that is exactly what they are, the natural numbers, but not in the base Ten, but in bases two thru sixteen. Therefore; please consider the following sequences (all are to 101 terms for consistency) for your upcoming second edition:
Base 2: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111, 100000, 100001, 100010, 100011, 100100, 100101, 100110, 100111, 101000, 101001, 101010, 101011, 101100, 101101, 101110, 101111, 110000, 110001, 110010, 110011, 110100, 110101, 110110, 110111, 111000, 111001, 111010, 111011, 111100, 111101, 111110, 111111, 1000000, 1000001, 1000010, 1000011, 1000100, 1000101, 1000110, 1000111, 1001000, 1001001, 1001010, 1001011, 1001100, 1001101, 1001110, 1001111, 1010000, 1010001, 1010010, 1010011, 1010100, 1010101, 1010110, 1010111, 1011000, 1011001, 1011010, 1011011, 1011100, 1011101, 1011110, 1011111, 1100000, 1100001, 1100010, 1100011, 1100100, 1100101, ...
Base 3: 1, 2, 10, 11, 12, 20, 21, 22, 100, 101, 102, 110, 111, 112, 120, 121, 122, 200, 201, 202, 210, 211, 212, 220, 221, 222, 1000, 1001, 1002, 1010, 1011, 1012, 1020, 1021, 1022, 1100, 1101, 1102, 1110, 1111, 1112, 1120, 1121, 1122, 1200, 1201, 1202, 1210, 1211, 1212, 1220, 1221, 1222, 2000, 2001, 2002, 2010, 2011, 2012, 2020, 2021, 2022, 2100, 2101, 2102, 2110, 2111, 2112, 2120, 2121, 2122, 2200, 2201, 2202, 2210, 2211, 2212, 2220, 2221, 2222, 10000, 10001, 10002, 10010, 10011, 10012, 10020, 10021, 10022, 10100, 10101, 10102, 10110, 10111, 10112, 10120, 10121, 10122, 10200, 10201, 10202, ...
Base 4: 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33, 100, 101, 102, 103, 110, 111, 112, 113, 120, 121, 122, 123, 130, 131, 132, 133, 200, 201, 202, 203, 210, 211, 212, 213, 220, 221, 222, 223, 230, 231, 232, 233, 300, 301, 302, 303, 310, 311, 312, 313, 320, 321, 322, 323, 330, 331, 332, 333, 1000, 1001, 1002, 1003, 1010, 1011, 1012, 1013, 1020, 1021, 1022, 1023, 1030, 1031, 1032, 1033, 1100, 1101, 1102, 1103, 1110, 1111, 1112, 1113, 1120, 1121, 1122, 1123, 1130, 1131, 1132, 1133, 1200, 1201, 1202, 1203, 1210, 1211, ...
Base 5: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 100, 101, 102, 103, 104, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 142, 143, 144, 200, 201, 202, 203, 204, 210, 211, 212, 213, 214, 220, 221, 222, 223, 224, 230, 231, 232, 233, 234, 240, 241, 242, 243, 244, 300, 301, 302, 303, 304, 310, 311, 312, 313, 314, 320, 321, 322, 323, 324, 330, 331, 332, 333, 334, 340, 341, 342, 343, 344, 400, 401, ...
Base 6: 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 55, 100, 101, 102, 103, 104, 105, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 200, 201, 202, 203, 204, 205, 210, 211, 212, 213,

214, 215, 220, 221, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, ...
Base 7: 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 160, 161, 162, 163, 164, 165, 166, 200, 201, 202, 203, ...
Base 8: 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, ...
Base 9: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, ...
Base 10: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, ...
Base A: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 8A, 90, 91, 92, ...
Base B: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, 7B, 80, 81, 82, 83, 84, 85, ...
Base C: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 6C, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, ...
Base D: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 3D, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 4D, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 5D, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 6C, 6D, 70, 71, 72, 73, ...
Base E: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 2E, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 3D, 3E, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 4D, 4E,

50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 5D, 5E, 60, 61, 62,
63, 64, 65, 66, 67, 68, 69, 6A, 6B, ...
Base F: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14,
15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26,
27, 28, 29, 2A, 2B, 2C, 2D, 2E, 2F, 30, 31, 32, 33, 34, 35, 36, 37, 38,
39, 3A, 3B, 3C, 3D, 3E, 3F, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A,
4B, 4C, 4D, 4E, 4F, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C,
5D, 5E, 5F, 60, 61, 62, 63, 64, 65, ...
Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

13 November 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
In the cited referenced text on page 82, under the recitation for the number eleven is the follow: "Given any 4 consecutive integers greater than 11, there is at least one of them that is divisible by a prime greater than 11." Question! For the other prime numbers, what is the number above which this still holds true? Please find the primes is order on the following page.
Reference: The Penguin Dictonary of Curious and Interesting Numbers, David Wells, Middlesex, England, 1986.
Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

2	0
3	1
5	3
7	7
11	9
13	63
17	63
19	168
23	322
29	322
31	1518
37	1518
41	1680
43	10,878
47	17,575
53	17,575
59	17,575
61	17,575
67	17,575
71	17,575
73	70,224
79	70,224
83	97,524
89	97,524
97	97,524
101	97,524
103	224,846
107	224,846
109	612,360
113	688,973
127	688,973
131	688,973
137	688,973
139	688,973
149	688,973

151
157
163
167
173
179
181
191
193
197
199
211
223
227
229

4 December 1992
Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
In the cited referenced text S12 on page 27, there is the following theorem numbered three: If n is a natural number > 2, then between n and n! there is at least one prime number. This beings up several questions. Only three will be addressed here. One, how many primes are there between n and n!? Or better yet, just how many primes are less than or equal to n factor-ial? Two, what is the last prime before n factorial? And three, what is the difference between n! and the Prime just preceeding it?

n	Last Prime before n!	the Diff.	Nbr of Primes < n!
1	0	0	0
2	0	0	0
3	5	1	3
4	23	1	9
5	113	7	30
6	719	1	128
7	5,039	1	675
8	40,289	31	4,231
9	362,867	13	30,969
10	3,628,789	11	258,689
11	39,916,787	13	2,428,956
12	479,001,599	1	25,306,287
13	6,227,020,777	23	289,620,751
14	87,178,291,199	1	

n	Last Prime before n!	the Diff.
15	1,307,674,367,953	47
16	20,922,789,887,947	53
17	355,687,428,095,941	59
18	6,402,373,705,727,959	41
19	121,645,100,408,831,899	101
20	2,432,902,008,176,639,969	31
21	51,090,942,171,709,439,969	31
22	1,124,000,727,777,607,679,927	73
23	25,852,016,738,884,976,639,911	89
24	620,448,401,733,239,439,359,927	73
25	15,511,210,043,330,985,983,999,851	149

From the above table, it is easily discernable that the prime quoted in column two is the nth prime as noted is column four. Therefore; for n>0 the following sequence for (n!) is : 0, 0, 1, 3, 9, 30, 128, 675, 4231, 30969, 258689, 2428956, 25306287, 289620751, ...
Two of the above sequences grow so quickly that they will exceed the two lines given in your forthcoming tome. However, the sequence of the differences is restated and continued here: 0, 0, 1, 1, 7, 1, 1, 31, 13, 11, 13, 1, 23, 1, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 1, 61, 1, 1, 193, 113, 127, 97, 1, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 1, 109, 163, 311, 173, 571, 271, 479, ...
The mirror image of the above sequence is the difference between n factorial

and the first prime that exceeds it. Keep in mind that 0!=1 by definition (Graham p.111). It is as follows: 1, 1, 1, 5, 7, 7, 11, 23, 17, 11, 1, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 1, 67, 223, 107, 127, 79, 37, 97, 61, 131, 1, 43, 97, 53, 1, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 1, 179, 229, 113, 1, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, ...

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One, the above two sequences contains just primes! These series are analogous to the "Fortunate" numbers, but I shall leave that sequence to a separate letter.

The difference between the two previous sequences is the "prime gap" surrounding the number n! The largest observed difference being about the number 71! and it is equal to 1608, which equals the spread of the pair (71! - 751, 71! + 857). Since the two sequences above contain only odd num-bers, the differences are always even, therefore; the following series is the difference divided by two: 1, 3, 7, 4, 6, 27, 15, 11, 7, 15, 45, 10, 45, 38, 45, 39, 95, 30, 31, 52, 93, 102, 95, 48, 22, 84, 127, 54, 94, 40, 19, 145, 87, 129, 49, 22, 85, 68, 66, 121, 90, 78, 146, 95, 156, 78, 71, 79, 225, 60, 65, 175, 66, 305, 192, 196, 215, 205, 420, 101, 186, 213, 160, 300, 132, 167, 117, 118, 804, 132, 187, 189, 198, 135, 246, 215, 264, 105, 392, 139, 255, 345, 257, 108, 639, 366, 153, 168, 581, 238, 125, 136, 328, 181, 270, 240, 337, 250, 309, ...

Not only is the result of the first difference above n! produce a prime, but so is the result of the difference between the n! and the second prime. They are as follows: 2, 3, 5, 7, 11, 13, 19, 31, 23, 19, 17, 43, 73, 41, 149, 41, 53, 61, 109, 37, 37, 71, 109, 193, 97, 173, 47, 101, 229, 163, 241, 83, 139, 103, 83, 577, 311, 47, 269, 61, 61, 107, 97, 89, 379, 149, 269, 83, 137, 167, 281, 89, 79, 443, 229, 157, 179, 563, 389, 277, 827, 281, 433, 151, 383, 1033, 79, 101, 137, 173, 971, 151, 79, 479, 449, 139, 149, 307, 839, 139, 757, 971, 227, 919, 229, 509, 593, 787, 1187, 1069, 251, 167, 193, 479, 557, 239, 257, 499, 109, 439, 233, ...

So is the result of the difference between the n! and the third prime after n! also produces a prime. They are as follows: 5, 7, 13, 17, 19, 37, 37, 31, 41, 19, 59, 109, 71, 179, 73, 59, 73, 113, 53, 47, 127, 149, 263, 107, 241, 59, 103, 317, 241, 317, 113, 197, 127, 109, 647, 397, 67, 281, 67, 211, 163, 109, 107, 439, 521, 709, 101, 383, 337, 397, 223, 337, 601, 281, 311, 389, 821, 421, 307, 911, 353, 787, 293, 431, 1051, 233, 461, 241, 307, 991, 251, 107, 487, 997, 151, 197, 647, 881, 157, 937, 1697, 487, 971, 311, 541, 1117, 1129, 1249, 1709, 727, 661, 257, 1153, 607, 461, 337, 523, 239, 547, 563, ...

What follows is a table of n! until the difference of the next prime becomes composite.

The nth term is compos:

1! : 1, 2, 4=2*2
2! : 1, 3, 5, 9=3*3
3! : 1, 5, 7, 11, 13, 17, 23, 25=5*5
4! : 5, 7, 13, 17, 19, 23, 29, 35=5*7
5! : 7, 11, 17, 19, 29, 31, 37, 43, 47, 53, 59, 61, 71, 73, 77=7*11
15
6! : 7, 13, 19, 23, 31, 37, 41, 49=7*7
7! : 11, 19, 37, 41, 47, 59, 61, 67, 73, 79, 107, 113, 127, 131, 139, 149, 157, 169=13*13
8! : 23, 31, 37, 41, 67, 103, 107, 109, 113, 139, 151, 163, 167, 173, 179, 187=11*17
9! : 17, 23, 31, 47, 61, 71, 73, 89, 97, 103, 107, 137, 139, 157,

163, 167, 179, 181, 187=11*17
10! : 11, 19, 41, 47, 53, 83, 97, 127, 131, 149, 167, 169=13*13 12
11! : 1, 17, 19, 29, 31, 37, 59, 109, 113, 131, 139, 149, 173, 179,
191, 211, 227, 239, 169=13*13
12! : 29, 43, 59, 83, 101, 137, 167, 191, 193, 197, 221=13*17 11
13! : 67, 73, 109, 139, 157, 181, 199, 289=17*17 8
14! : 19, 41, 71, 79, 103, 127, 167, 193, 229, 353, 361=19*19 11
15! : 43, 149, 179, 223, 227, 361=19*19
16! : 23, 41, 73, 149, 163, 173, 197, 199, 233, 277, 283, 323=17*19 12
17! : 31, 53, 59, 83, 109, 167, 181, 263, 281, 283, 293, 307, 349,
367, 397, 421, 487, 599, 631, 739, 761, 769, 779=19*41 23
18! : 37, 61, 73, 101, 103, 107, 137, 173, 179, 239, 241, 257, 271,
293, 331, 337, 383, 397, 463, 541, 613, 617, 683, 713=23*31 24
19! : 89, 109, 113, 137, 139, 241, 281, 311, 389, 463, 467, 641,
659, 701, 713=23*31
20! : 29, 37, 53, 79, 89, 137, 139, 157, 167, 211, 271, 277, 283,
353, 367, 379, 439, 491, 571, 617, 619, 661, 673, 743, 839,
919, 941, 953, 997, 1091, 1189=29*41 31

The above table suggests yet an other sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows:
3, 4, 8, 8, 15, 8, 18, 16, 19, 12, 19, 11, 8, 11, 6, 12, 23, 24, 15, 31,
... .

Just like the preceeding sequences on the side greater that the factorial of n, so do we have them on the lesser side as well. Not only is the result of the first difference produce a prime, but so is the result of the difference between the n! and the second preceeding prime. They are as follows: 3, 5, 11, 11, 17, 37, 17, 17, 17, 13, 61, 17, 59, 71, 61, 43, 113, 71, ...
.

So is the result of the difference between the n! and the third prime also produces a prime, beginning with n=4. They are as follows: 7, 13, 19, 19, 43, 29, 23, 43, 17, 67, 43, 71, 89, 239, 47, 197, 151, ...

What follows is a table of n! until the difference of the previous prime becomes composite.

The nth term is compos:

1! :
2! :
3! : 1, 3, 4=2*2
4! : 1, 5, 7, 11, 13, 17, 19, 21=3*7
5! : 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 49=7*7 12
6! : 1, 11, 19, 29, 37, 43, 47, 59, 61, 67, 73, 77=7*7
7! : 1, 17, 19, 29, 31, 37, 41, 47, 53, 67, 71, 73, 83, 89, 97,
103, 107, 109, 121=11*11
8! : 31, 37, 43, 67, 79, 83, 89, 107, 127, 131, 143=11*13 11
9! : 13, 17, 29, 79, 121=11*11 9
10! : 11, 17, 23, 37, 41, 53, 89, 103, 121=11*11
11! : 13, 17, 43, 73, 83, 89, 97, 113, 149, 151, 163, 167, 199,
247=13*19 10
12! : 1, 13, 17, 37, 61, 73, 83, 107, 139, 169=13*13
13! : 23, 61, 67, 89, 101, 103, 109, 163, 191, 193, 223, 227, 233,
269, 281, 283, 289=17*17
14! : 1, 17, 43, 47, 61, 67, 89, 131, 197, 199, 211, 227, 233, 313,
347, 353, 359, 373, 419, 449, 461, 499, 601, 607, 659, 667=23*29 26
15! : 47, 59, 71, 79, 89, 97, 139, 157, 163, 167, 227, 251, 257, 293,
313, 317, 347, 367, 401, 419, 449, 461, 463, 467, 493=17*29 25
16! : 53, 71, 89, 107, 139, 163, 181, 227, 233, 239, 263, 307, 317,
337, 353, 359, 367, 421, 431, 433, 449, 457, 467, 563, 569, 571,
587, 601, 641, 673, 677, 701, 739, 743, 757, 787, 911, 947, 967,

983, 1007=19*53
17! : 59, 61, 239, 281, 307, 367, 373, 389, 401, 487, 491, 547, 587,
619, 631, 653, 673, 719, 743, 751, 811, 821, 823, 827, 839, 859,
883, 941, 947, 1009, 1151, 1163, 1201, 1271=31*41 34
18! : 41, 43, 47, 61, 101, 109, 131, 139, 157, 193, 257, 283, 311,
317, 331, 349, 421, 431, 439, 457, 499, 547, 557, 557, 589=19*31 25
19! : 101, 113, 197, 199, 251, 263, 307, 331, 353, 359, 409, 541,
659, 887, 929, 961=31*31
20! : 31, 71, 151, 163, 179, 191, 251, 257, 269, 353, 367, 397, 433,
443, 521, 601, 613, 631, 641, 659, 661, 797, 823, 853, 857, 859,
863, 877, 1019, 1063, 1097, 1109, 1171, 1181, 1213, 1229, 1279,
1291, 1297, 1361, 1423, 1427, 1493, 1537=29*53 44

The above table suggests yet an other sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 0, 0, 3, 8, 11, 12, 19, 11, 5, 9, 14, 10, 17, 26, 25, 41, 34, 25, 16, 44, ...

Given the sequence immediately above and its conterpart cited earlier, the sum of the two less the two end points states the number of prime differences surrounding n factorial. It is as follows: 2, 3, 9, 14, 24, 18, 35, 25, 22, 19, 31, 19, 23, 35, 29, 51, 55, 47, 29, 73, ...

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One, for n>2, there exists at least six primes, three below n! and three above n! which represent the differences of the consecutive primes and n! Example, for n=10, there are 3 consecutive primes immediately below 10!, they being 3628777, 3628783 and 3628789, and 3 consecutive primes immediately above 10!, they being 3628811, 3628819 and 3628841. The differeces are -23, -17, -11, +11, +19 and +41.

Reference: S12: A Selection Of Problems in the Theory Of Numbers, Waclaw Sierpinski (1882-1969), A Pergamon Press Book, the MacMillan Co,. New York, New York, 1964.
KN1: Seminumerical Algorithms, "The Art of Computer Programming," Ed. 2, Vol. 2, Page 365, Donald Ervin Knuth, Addison-Wesley Publ. Co., Reading, Mass, 1980.
Concrete Mathematics, A Foundation For Computer Science, Ronald Lewis Graham, Donald Ervin Knuth, Oren Patashnik, Addison-Wesley Publishing Co., Reading, Mass, 1989.
Stephen Wolfram "Mathematica," MS-DOS 386/7 Version 2.0.2 released June 1991, the function PrimePi[n!].

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

13 July 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following thirty-some odd sequences, most to the first 101 terms for inclusion in your second edition of the above. "It is easy to prove that for any given natural number k the equation I (n+k) = I (n) has at least one solution in the natural numbers n." page 231. There-fore the following sequence: 1, 4, 3, 8, 5, 24, 5, 13, 9, 20, 7, 48, 13, 16,
13, 26, 17, 52, 19, 37, 21, 44, 13, 96, 25, 34, 27, 32, 13, 124, 17, 52, 33, 41, 19, 104, 35, 52, 37, 65, 25, 123, 17, 73, 39, 92, 41, 183, 35, 76, 39, 68, 53, 156, 35, 64, 57, 116, 41, 248, 61, 73, 61, 104, 65, 144, 67, 82, 41, 140, 37, 208, 73, 124, 65, 104, 37, 267, 65, 109, 81, 143, 83, 241, 85, 148, 87, 143, 37, 365, 41, 184, 61, 188, 55, 219, 97, 97, 91, 152, 101, ...
" ... have proved that for every natural number k<2*10^58 the equation I (n+k) = I (n) has at least two solutions in natural numbers n." page 232. Therefore the following sequence: 3, 7, 5, 14, 9, 34, 7, 16, 15, 26, 11, 68,
39, 28, 15, 32, 33, 72, 25, 40, 35, 56, 17, 101, 45, 37, 45, 56, 29, 152, 31, 61, 39, 56, 35, 144, 37, 61, 39, 74, 41, 128, 35, 88, 45, 161, 47, 192, 49, 82, 51, 74, 95, 216, 43, 97, 75, 203, 59, 304, 91, 88, 63, 122, 117, 194, 129, 112, 51, 146, 71, 288, 117, 148, 73, 119, 55, 292, 73, 130, 135, 146, 225, 246, 133, 172, 95, 146, 89, 372, 65, 259, 93, 194, 89, 339, 123, 112, 99, 164, 143, ...
The difference between the above two series is the following sequence:

2, 3, 2, 6, 4, 10, 2, 3, 6, 6, 4, 20, 26, 12, 2, 6, 16, 20, 6, 3, 14, 12, 4, 5, 20, 3, 18, 24, 16, 28, 14, 9, 6, 15, 16, 40, 2, 9, 2, 9, 16, 5, 18, 15, 6, 69, 6, 9, 14, 6, 12, 6, 42, 60, 8, 33, 18, 87, 18, 56, 30, 15, 2, 18, 52, 50, 62, 30, 10, 6, 34, 80, 44, 24, 8, 15, 18, 25, 8, 21, 54, 3, 142, 5, 48, 24, 8, 3, 52, 7, 24, 75, 32, 6, 34, 120, 26, 15, 8, 12, 42, ...

As long as we are on this train of thought, then the next logical

sequence is the third occurrence, and it is as follows: 15, 8, (3540000),

16, 15, 36, 21, 19, (100000), 35, 27, 72, 51, 34, 17, 38, 35, 73, 57, 52, (100000), 73, 23, 109, 75, 52, 55, 68, 51, 180, 39, 64, 45, 68, 75, 146, 49, 64, 45, 80, 111, 148, 43, 91, 51, 182, 65, 202, 147, 100, 57, 104, 123, 219, 55, 112, 91, 232, 177, 325, 93, 109, 105, 128, 183, 219, 201, 136, 57, 152, 111, 292, 175, 238, 75, 122, 77, 312, 79, 148, 145, 152, 243, 256, 153, 194, 99, 176, 119, 386, 91, 322, 135, 329, 95, 366, 273, 178, 117, 185, 237, .. .
Continuing the next sequence is the fourth occurrence, and it is as

follows: 104, 10, (3540000), 20, 21, 39, 45, 25, (100000), 100, 33, 78, 63,

41, 21, 50, 39, 82, 225, 55, (100000), 77, 69, 111, 99, 89, (100000), 82, 87, 194, 93, 76, 55, 74, 105, 164, 111, 73, 65, 95, 123, 153, 85, 112, 55, 184, 77, 218, 315, 130, 63, 178, 159, 246, 91, 133, 95, 266, 357, 360, 183, 124, 115, 152, 195, 244, 429, 148, 69, 155, 153, 303, 219, 259, 85, 128, 123, 327, 111, 157, 165, 164, 249, 296, 195, 247, 135, 182, 155, 456, 273, 353, 155, 365, 171, 369, 291, 181, 135, 200, 267, ...
And the final sequence is the fifth occurrence, and it is as follows:

164, 26, (3540000), 35, 15556, 43, 75, 28, (100000), 130, 45, 86, 75, 56, (100000), 56, 51, 102, 273, 70, (100000), 80, 99, 136, 105, 91, (100000), 112, 93, 208, 117, 100, (50000), 119, 111, 181, 399, 76, 105, 104, 153, 168, 129, 146, 63, 200, 135, 222, 525, 175, 85, 182, 429, 268, 99, 136, (50000), 290, 429, 369, 207, 190, (50000), 200, 255, 264, 441, 169, 115, 170, 213, 327, 651, 281, 105, 146, 125, 372, 231, 160, (50000), 224, 261, 306, 219, 286, 145, 185, 267, 482, 357, 364, 195, 376, 285, 384, 385, 193, 165, 260, 303, ...
In the same vein but a different function 'Sum of the Divisors' the

following sequence is the first occurrence for which o (n+k)= o (n): 14, 33,

382, 51, 6, 20, 10, 15, 14, 21, 28, 35, 182, 24, 26, 30, 142, 40, 34, 42, 20, 57, 135, 70, 30, 99, 42, 66, 406, 88, 56, 60, 54, 93, 24, 105, 248, 147, 44, 63, 30, 80, 435, 114, 52, 196, 310, 140, 40, 105, 92, 160, 66, 120, 140, 105, 88, 352, 154, 224, 118, 177, 60, 117, 78, 220, 182, 135, 8786, 96, 112, 210, 752, 135, 92, 294, 110, 365, 735, 126, 126, 204, 60, 270, 102, 105, 254, 165, 78, 264, 88, 195, 174, 440, 114, 280, 138, 168, 124, 210, 316, ...
For the second occurrance the following sequence: 206, 54, 11935, 66,

46, 155, 62, 69, 16, 174, 154, 104, 782, 33, 62, 55, 238, 60, 158, 51, 30, 85, 231, 143, 46, 150, 48, 159, 496, 161, 58, 110, 35562, 96, 42, 130, 302, 246, 56, 84, 54, 135, 602, 123, 70, 205, 658, 165, 66, 132, 158, 198, 406, 180, 166, 132, 102, 852, 376, 315, 188, 224, 76, 120, 526, 232, 795, 186, 24885, 120, 945, 260, 862, 280, 130, 352, 190, 459, 1034, 147, 144, 748, 184, 370, 166, 390, 358, 228, 114, 352, 130, 267, 11842, 736, 170, 330, 686, 231, 154, 255, 658, ...
And the difference between the two produces the following sequence:

192, 21, 11553, 15, 40, 135, 52, 54, 2, 153, 126, 69, 600, 9, 36, 25, 96, 20, 124, 9, 10, 28, 96, 73, 16, 51, 6, 93, 90, 73, 2, 50, 35508, 3, 18, 25, 54, 99, 12, 21, 24, 55, 167, 9, 18, 9, 348, 25, 26, 27, 66, 38, 340, 60, 26, 27, 14, 500, 222, 91, 70, 47, 16, 3, 448, 12, 613, 51, 16099, 24, 833, 50, 110, 145, 38, 58, 80, 94, 299, 21, 18, 544, 124, 100, 64, 285, 104, 63, 36, 88, 42, 72, 11668, 296, 56, 50, 548, 63, 30, 45, 342, ...
On page 234, "for every natural number s there exists a natural number m such that the equation I (n) = m has precisely s solutions in natural numbers. We do not know the answer to this question even in the simple case of s=1. ... As was show by V. L. Klee Jr. [3], there are no such numbers m < 10^400." Restating the original series and then continuing it, I present: "1, 2, 4, 8, 12, 32, 36, 40, 24, 48, 160, 396, 2268, 704,"
312, 72, 336, 216, 936, 144, 624, 1056, 1760, 360, 2560, 384, 288, 1320, 3696, 240, 768, 9000, 432, 7128, 4200, 480, 576, 1296, 1200, 15936, 3312, 3072, 3240, 864, 3120, 7344, 3888, 7220, 1680, 4992, 17640, 2016, 1152, 6000, 12288, 4752, 2688, 3024, 13680, 9984, 1728, 1920, 2400, 7560, 2304, 22848, 8400, 29160, 5376, 3360, 1440, 13248, 11040, 27720, 21840, 9072,

38640, 9360, 81216, 4032, 5280, 4800, 4608, 16896, 3456, 3840, 10800, 9504, 18000, 23520, 39936, 5040, 26208, 27360, 6480, 9216, 2880, 26496, 34272, 23328, 28080, ...
However, just as there are solutions for I (n) = m, so are there values of m to which s = 0 or restated, there are no solutions in m. Beiler, page 91. They begin: 14, 26, 34, 38, 50, 62, 68, 74, 76, 86, 90, 94, 98, 114,
118, 122, 124, 134, 142, 146, 152, 154, 158, 170, 174, 182, 186, 188, 194, 202, 206, 214, 218, 230, 234, 236, 242, 244, 246, 248, 254, 258, 266, 274, 278, 284, 286, 290, 298, 302, 304, 308, 314, 318, 322, 326, 334, 338, 340, 350, 354, 362, 364, 370, 374, 376, 386, 390, 394, 398, 402, 404, 406, 410, 412, 414, 422, 426, 428, 434, 436, 446, 450, 454, 458, 470, 472, 474, 482, 484, 488, 494, 496, 510, 514, 516, 518, 526, 530, 532, 534, ...
Or if you like the above divided by two so as to save some room: 7, 13,

17, 19, 25, 31, 34, 37, 38, 43, 45, 47, 49, 57, 59, 61, 62, 67, 71, 73, 76, 77, 79, 85, 87, 91, 93, 94, 97, 101, 103, 107, 109, 115, 117, 118, 121, 122, 123, 124, 127, 129, 133, 137, 139, 142, 143, 145, 149, 151, 152, 154, 157, 159, 161, 163, 167, 169, 170, 175, 177, 181, 182, 185, 187, 188, 193, 195, 197, 199, 201, 202, 203, 205, 206, 207, 211, 213, 214, 217, 218, 223, 225, 227, 229, 235, 236, 237, 241, 242, 244, 247, 248, 255, 257, 258, 259, 263, 265, 266, 267, 269, ...
The equation I (n) = m has just two solutions: 1, 10, 22, 28, 30, 46,

52, 54, 58, 66, 70, 78, 82, 102, 106, 110, 126, 130, 136, 138, 148, 150, 166, 172, 178, 190, 196, 198, 210, 222, 226, 228, 238, 250, 262, 268, 270, 282, 292, 294, 306, 310, 316, 330, 342, 346, 358, 366, 372, 378, 382, 388, 418, 430, 438, 442, 462, 466, 478, 490, 498, 502, 506, 508, 522, 546, 556, 562, 568, 570, 580, 586, 598, 606, 618, 630, 642, 646, 652, 658, 676, 682, 690, 708, 718, 726, 738, 742, 750, 772, 786, 796, 808, 810, 812, 822, 826, 838, 852, 856, 858, ...
The equation I (n) = m has just three solutions: 2, 44, 56, 92, 104,

116, 140, 164, 204, 212, 260, 296, 332, 344, 356, 380, 392, 444, 452, 476, 524, 536, 564, 584, 588, 620, 632, 684, 692, 716, 744, 764, 776, 836, 860, 884, 932, 956, 980, 1004, 1016, 1112, 1124, 1136, 1172, 1196, 1284, 1292, 1304, 1316, 1352, 1364, 1416, 1436, 1484, 1544, 1592, 1616, 1644, 1652, 1676, 1704, 1712, 1724, 1772, 1812, 1820, 1880, 1892, 1940, 1952, 1964, 2036, 2060, 2124, 2172, 2180, 2192, 2204, 2216, 2288, 2300, 2324, 2360, 2372, 2384, 2432, 2444, 2456, 2516, 2564, 2604, 2612, 2636, 2732, 2744, 2844, 2852, 2876, 2892, 2900, ...
The equation I (n) = m has just four solutions: 4, 6, 18, 42, 100, 162,

184, 208, 328, 424, 460, 468, 486, 492, 616, 636, 664, 688, 700, 712, 784, 820, 900, 904, 1020, 1060, 1072, 1168, 1240, 1264, 1276, 1288, 1300, 1356, 1360, 1384, 1404, 1458, 1480, 1528, 1672, 1740, 1768, 1864, 1896, 1900, 1908, 2008, 2028, 2032, 2148, 2196, 2220, 2224, 2248, 2296, 2328, 2332, 2344, 2380, 2500, 2508, 2568, 2584, 2620, 2628, 2704, 2860, 2868, 2872, 3012, 3180, 3184, 3204, 3220, 3232, 3256, 3288, 3304, 3352, 3424, 3460, 3544, 3580, 3624, 3820, 3904, 3912, 3916, 3948, 4068, 4120, 4180, 4308, 4344, 4360, 4384, 4420, 4422, 4432, 4632, ...
The equation I (n) = m has just five solutions: 8, 20, 220, 272, 300,

368, 416, 456, 500, 656, 732, 848, 876, 1092, 1160, 1212, 1236, 1328, 1376, 1424, 1568, 1624, 1716, 1808, 2144, 2244, 2336, 2420, 2460, 2480, 2528, 2556, 2768, 3056, 3080, 3252, 3320, 3344, 3536, 3560, 3612, 3728, 3732, 3900, 4016, 4020, 4064, 4260, 4448, 4496, 4520, 4688, 4692, 5100, 5168, 5232, 5340, 5360, 5408, 5512, 5744, 5840, 5984, 6036, 6132, 6156, 6200, 6320, 6368, 6380, 6464, 6608, 6636, 6704, 6848, 7088, 7212, 7248, 7536, 7700, 7808, 7932, 8004, 8120, 8240, 8600, 8720, 8768, 8864, 9012, 9276, 9320, 9488, 9536, 9560, 9728, 9800, 9824, 9940, ...

On page 235, "It is not known whether for every natural number k there exists a natural number m for which the equation o (x) = m has precisely k solutions in natural numbers x. This follows from the conjecture H (...). It can be proved that if m denotes the least of the numbers for which o (x) = m has precisely k solutions, then" : "1, 12, 24, 96, 72, 168, 240, 432, 360, 504, 576, 1512, 1080, 1008, 720, 2304, 3600, 5376, 2160, 1440," But this quoted series is incorrect. The problem with the above sequence is not that 432 does not have eight solutions for o (n) = 432 (they being 230, 238, 255, 321, 355, 371, 391 & 431.), but that 432 is not the first number to possess this trait. The number 336 is, with the eight solutions being 132, 140, 182, 188, 195, 249, 287 & 299. Also 2520 is the number with 19 solutions and both 2160 and 1440 have one more solution that they are credited with, although they still retain the distinction of being the first. (2160 has as its 20 solutions: 870, 918, 920, 952, 1074, 1246, 1298, 1334, 1335, 1431, 1438, 1479, 1595, 1615, 1795, 1883, 1969, 2033, 2047 & 2059.) (1440 has as its 21 solutions: 552, 570, 594, 616, 790, 826, 874, 885, 957, 958, 969, 1015, 1045, 1077, 1195, 1253, 1343, 1349, 1357, 1363 & 1439.) Let me restate the series correctly from the beginning:

1, 12, 24, 96, 72, 168, 240, 336, 360, 504, 576, 1512, 1080, 1008, 720, 2304, 3600, 5376, 2520, 2160, 1440, 10416, 13392, 3360, 4032, 3024, 7056, 6720, 2880, 6480, 10800, 13104, 5040, 6048, 4320, 13440, 5760, 18720, 20736, 19152, 22680, 43680, 28080, 26208, 14400, 16128, 25200, 11520, 8640, 78120, 18144, 21600, 62208, 35280, 97200, 62496, 142848, 10080, 15120, 55440, 44640, 66960, 38880, 24192, 42336, 98496, 52416, 17280, 97920, 64512, 46080, 63360, 123120, 25920, 54720, 117936, 231840, 45360, 20160, 127680, 57600, 43200, 75600, 200880, 48384, 228096, 158400, 147840, 131328, 215040, 334800, 275184, 172368, 196992, 133920, 142560, 34560, 30240, 368640, 72576, (392212), ...

However, just as there are solutions for o (n) = m, so are there values of m to which s = 0 or restated, there are no solutions in m. They begin:

2, 5, 9, 10, 11, 16, 17, 19, 21, 22, 23, 25, 26, 27, 29, 33, 34, 35, 37, 41, 43, 45, 46, 47, 49, 50, 51, 52, 53, 55, 58, 59, 61, 64, 65, 66, 67, 69, 70, 71, 73, 75, 76, 77, 79, 81, 82, 83, 85, 86, 87, 88, 89, 92, 94, 95, 97, 99, 100, 101, 103, 105, 106, 107, 109, 111, 113, 115, 116, 117, 118, 119, 122, 123, 125, 129, 130, 131, 134, 135, 136, 137, 139, 141, 142, 143, 145, 146, 147, 148, 149, 151, 153, 154, 155, 157, 159, 161, 163, 165, 166, ...

The equation o (n) = m has just one solutions: 1, 3, 4, 6, 7, 8, 13,

14, 15, 20, 28, 30, 36, 38, 39, 40, 44, 57, 62, 63, 68, 74, 78, 91, 93, 102, 110, 112, 121, 127, 133, 138, 150, 158, 160, 162, 164, 171, 174, 176, 183, 194, 195, 198, 200, 204, 212, 217, 222, 230, 242, 255, 256, 258, 260, 266, 278, 282, 284, 296, 300, 304, 306, 307, 314, 318, 330, 332, 338, 348, 350, 352, 354, 363, 364, 368, 374, 380, 381, 396, 398, 400, 402, 410, 414, 422, 458, 462, 464, 465, 474, 476, 488, 494, 496, 500, 508, 510, 511, 512, 518, ...

The equation o (n) = m has just two solutions: 12, 18, 31, 32, 54, 56,

80, 98, 104, 108, 114, 124, 126, 128, 132, 140, 152, 156, 182, 186, 210, 264, 272, 280, 308, 320, 342, 378, 390, 392, 399, 403, 408, 416, 440, 444, 448, 492, 522, 532, 570, 572, 594, 608, 630, 632, 726, 762, 770, 774, 780, 784, 800, 828, 868, 880, 884, 900, 920, 924, 942, 948, 954, 984, 1014, 1024, 1026, 1032, 1040, 1044, 1062, 1088, 1098, 1110, 1164, 1178, 1188, 1194, 1218, 1230, 1272, 1280, 1328, 1350, 1352, 1364, 1374, 1386, 1408, 1428, 1430, 1472, 1484, 1500, 1520, 1568, 1572, 1608, 1610, 1656, 1664, ...

The equation o (n) = m has just three solutions: 24, 42, 48, 60, 84,

90, 224, 228, 234, 248, 270, 294, 324, 450, 468, 528, 558, 620, 640, 660, 810, 882, 888, 896, 968, 972, 1020, 1050, 1104, 1116, 1140, 1216, 1232, 1240, 1274, 1332, 1392, 1400, 1452, 1456, 1464, 1482, 1524, 1530, 1600, 1694, 1716, 1760, 1890, 1896, 1932, 1960, 1968, 2028, 2128, 2176, 2256, 2286, 2294, 2418, 2436, 2460, 2464, 2484, 2660, 2772, 2964, 3042, 3132, 3280, 3294, 3328, 3384, 3408, 3584, 3684, 3724, 3808, 3852, 3864, 3876, 3912, 3924, 3948, 3984, 3990, 4160, 4230, 4248, 4260, 4290, 4298, 4312, 4446, 4452, 4488, 4576, 4776, 4824, 4944, 4968, ...

The equation o (n) = m has just four solutions: 96, 120, 180, 312, 372,

420, 434, 456, 540, 546, 560, 624, 702, 728, 798, 816, 930, 1064, 1120, 1170, 1404, 1632, 1638, 1674, 1710, 1776, 1792, 1944, 2100, 2240, 2544, 2560, 2664, 2760, 2800, 2844, 2856, 2940, 2952, 3000, 3040, 3048, 3060, 3080, 3096, 3108, 3224, 3432, 3492, 3510, 3564, 3768, 3822, 3920, 4004, 4140, 4356, 4424, 4572, 4644, 4650, 4656, 4712, 4836, 4914, 5004, 5088, 5120, 5130, 5320, 5496, 5568, 5640, 5652, 5670, 5724, 5832, 6200, 6288, 6400, 6510, 6672, 6776, 6858, 6960, 7224, 7280, 7360, 7448, 7524, 7536, 7650, 7688, 7704, 7872, 7944, 7968, 8060, 8244, 8256, 8460, ...

The equation o (n) = m has just five solutions: 72, 144, 192, 216, 588,

600, 648, 792, 936, 992, 1056, 1224, 1302, 1320, 1560, 1736, 1980, 2040, 2088, 2112, 2268, 2448, 2730, 2790, 2912, 3038, 3136, 3312, 3472, 3520, 3534, 3552, 3672, 3792, 3816, 3936, 4056, 4092, 4340, 4440, 4864, 4872, 4920, 4960, 5082, 5334, 5600, 5796, 5904, 5940, 6096, 6156, 6768, 6936, 7168, 7368, 7380, 7800, 7936, 8148, 8280, 8320, 8432, 8580, 8664, 8704, 8856, 8904, 9180, 9312, 9432, 9552, 9648, 9660, 9768, 9900, 9920, 10032, 10200, 10240, 10248, 10320, 10530, 10602, 10692, 10980, 10992, 11016, 11136, 11256, 11400, 11440, 11700, 11844, 11928, 12012, 12152, 12192, 12264, 12400, 12648, ...

The following is the first occurrence for n when I (n) = k/2 inwhich n is not a prime one less than n: 4, 8, 9, 15, 22, 21, 0, 32, 27, 25, 46, 35, 0, 58, 62, 51, 0, 57, 0, 55, 49, 69, 94, 65, 0, 106, 81, 87, 118, 77, 0, 85, 134, 0, 142, 91, 0, 0, 158, 123, 166, 129, 0, 115, 0, 141, 0, 119, 0, 125, 206, 159, 214, 133, 121, 145, 0, 177, 0, 143, 0, 0, 254, 255, 262, 161, 0, 274, 278, 213, 0, 185, 0, 298, 302, 0, 0, 169, 0, 187, 243, 249, 334, 203, 0, 346, 0, 267, 358, 209, 0, 235, 0, 0, 382, 221, 0, 394, 398, 275, 0, ...

The following is the last occurrence for n when I (n) = k/2: 2, 6, 12, 18, 30, 22, 42, 0, 60, 54, 66, 46, 90, 0, 58, 62, 120, 0, 126, 0, 150, 98, 138, 94, 210, 0, 106, 162, 174, 118, 198, 0, 240, 134, 0, 142, 270, 0, 0, 158, 330, 166, 294, 0, 276, 0, 282, 0, 420, 0, 250, 206, 318, 214, 378, 242, 348, 0, 354, 0, 462, 0, 0, 254, 510, 262, 414, 0, 274, 278, 426, 0, 630, 0, 298, 302, 0, 0, 474, 0, 660, 486, 498, 334, 588, 0, 346, 0, 690, 358, 594, 0, 564, 0, 0, 382, 840, 0, 394, 398, 750, 0, ...

(In the preceeding two sequences, the series of occurrences of Zeroes matches an earlier sequence presented at the bottom of page 3.)

The following is the first occurrence for n when o (n) = k: 1, 0, 2, 3, 0, 5, 4, 7, 0, 0, 0, 6, 9, 13, 8, 0, 0, 10, 0, 19, 0, 0, 0, 14, 0, 0, 0, 12, 0, 29, 16, 21, 0, 0, 0, 22, 0, 37, 18, 27, 0, 20, 0, 43, 0, 0, 0, 33, 0, 0, 0, 0, 0, 34, 0, 28, 49, 0, 0, 24, 0, 61, 32, 0, 0, 0, 0, 67, 0, 0, 0, 30, 0, 73, 0, 0, 0, 45, 0, 57, 0, 0, 0, 44, 0, 0, 0, 0, 0, 40, 36, 0,

50, 0, 0, 42, 0, 52, 0, 0, 0, ...
The following is the first occurrence for n when o (n) = k less the Zeroes:
1, 2, 3, 5, 4, 7, 6, 9, 13, 8, 10, 19, 14, 12, 29, 16, 21, 22, 37,
18, 27, 20, 43, 33, 34, 28, 49, 24, 61, 32, 67, 30, 73, 45, 57, 44, 40,
36, 50, 42, 52, 101, 63, 85, 109, 91, 74, 54, 81, 48, 68, 64, 93, 86, 121,
137, 76, 66, 149, 111, 99, 157, 133, 106, 163, 60, 98, 173, 129, 88, 117,
169, 80, 105, 193, 72, 197, 199, 134, 104, 211, 102, 100, 146, 84, 147,
229, 90, 114, 241, 112, 96, 128, 217, 257, 171, 215, 148, 136, 201, 277,
... .
To expand on your Sequence Nbr. 1215, I (n) = I (n+1) : "1, 3, 15, 104,
164, 194, 255, 495, 584, 975, 2204, 2625, 2834, 3255, 3705, 5186, 5187,"
10604, 11714, 13365, 18315, 22935, 25545, 32864, 38804, 39524, 46215, 48704,
49215, 49335, 56864, 57584, 57645, 64004, 65535, 73124, 105524, 107864,
123824, 131144, 164175, 184635, 198315, 214334, 215775, 256274, 286995,
307395, 319275, 347324, 388245, 397485, 407924, 415275, 454124, 491535,
524432, 525986, 546272, 568815, 589407, 679496, 686985, 840255, 914175,
936494, 952575, 983775, 1025504, 1091684, 1231424, 1259642, 1276904, 1390724,
1405845, 1574727, 1659585, 1759874, 1788254, 1925564, 2123583, 2200694,
2388044, 2521694, 2539004, 2619705, 2648204, 2759925, 2792144, 2822715,
2847584, 3104744, 3137355, 3170936, 3240614, 3289934, 3653564, 3693525,
3794834, 3877184, 3988424, 4002405, 4034744, ...
To expand on your Sequence Number 1328, I (n) = I (n+2) : "1, 4, 7, 8,

10, 26, 32, 70, 74, 122, 146, 308, 314, 386, 512, 554, 572, 626, 635, 728,
794, 842, 910, 914, 1015, 1082,"
1226, 1322, 1330, 1346, 1466, 1514, 1608, 1754, 1994, 2132, 2170, 2186,
2306, 2402, 2426, 2474, 2590, 2642, 2695, 2762, 2906, 3242, 3314, 3506,
3746, 3866, 3986, 4034, 4274, 4292, 4338, 4682, 4946, 5114, 5186, 5594,
5714, 5834, 5950, 6122, 6434, 6497, 6506, 6626, 6764, 7034, 7466, 8042,
8114, 8354, 8522, 8546, 8714, 8882, 9100, 9122, 9242, 9758, 9866, 10154,
10202, 10226, 10307, 10466, 10826, 10874, 11162, 11402, 12074, 12146, 12212,
12242, 12266, 12317, 12434, ...
As long as we are on this train of thought, then the next logical sequence
is to include the occurrences of n when o (n+1) = o (n), and it is as follows:
14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364, 14841, 18873,
19358, 20145, 24957, 33998, 36566, 42818, 56564, 64665, 74918, 79826, 79833,
84134, 92685, 109214, 111506, 116937, 122073, 138237, 147454, 161001, 162602,
166934, 174717, 190773, 193893, 201597, 230390, 274533, 289454, 347738,
383594, 416577, 422073, 430137, 438993, 440013, 445874, 455373, 484173,
522621, 544334, 605985, 621027, 649154, 655005, 685995, 695313, 739556,
792855, 937425, 949634, 1154174, 1174305, 1187361, 1207358, 1238965, 1642154,
1670955, 1765664, 1857513, 2168906, 2284814, 2305557, 2913105, 3296864,
3477435, 3571905, 3582224, 3682622, 3726009, 4328937, 4473782, 4481985,
4701537, 4795155, 5002335, 5003738, 5181045, 5351175, 5446425, 5459024,
5517458, 6309387, 6431732, 6444873, 6514995, 6771405, 7192917, 7263944,
7796438, 7845386, 7955492, ...
Continuing is to include the occurrences of n when o (n+2) = o (n), and
it is as follows: 33, 54, 284, 366, 834, 848, 918, 1240, 1504, 2910, 2913,
3304, 4148, 4187, 6110, 6902, 7169, 7912, 9359, 10250, 10540, 12565, 15085,
17272, 17814, 19004, 19688, 21410, 21461, 24881, 25019, 26609, 28124, 30592,
30788, 31484, 38210, 38982, 39786, 40310, 45354, 46863, 49225, 51835, 53106,
53963, 55286, 59987, 76360, 77057, 81055, 83094, 94996, 95392, 96728, 101101,
117570, 117858, 121394, 124758, 127585, 143369, 147340, 149149, 149750,
150419, 163936, 167560, 170114, 170561, 173920, 175796, 181384, 197260,
205727, 215069, 220817, 239954, 278920, 280787, 292315, 293656, 319955,
334540, 334983, 336505, 344416, 359454, 360325, 360685, 370435, 388074,

418307, 434433, 463218, 472323, 477904, 510340, 516026, 543453, 564857, ...

From page 235, the first occurrence of k when n - I (n) = k, and it is as follows: 3, 4, 9, 6, 25, 10, 15, 12, 21, 0, 35, 18, 33, 26, 39, 24, 65, 34, 51, 38, 45, 30, 95, 36, 69, 0, 63, 52, 161, 42, 87, 48, 93, 0, 75, 54, 217, 74, 99, 76, 185, 82, 123, 60, 117, 66, 215, 72, 141, 0, 235, 0, 329, 78, 159, 98, 105, 0, 371, 84, 177, 122, 135, 96, 305, 90, 427, 134, 201, 102, 335, 108, 213, 146, 207, 148, 245, 114, 511, 152, 189, 130, 395, 164, 165, 0, 415, 120, 581, 126, 267, 132, 261, 138, 623, 144, 1501, 194, 195, 0, 485, ...

The sequence of k's when n - I (n) = k has no solutions (the Zeros above), and it is as follows: 10, 26, 34, 50, 52, 58, 86, 100, 116, 122, 130, 134, 146, 154, 170, 172, 186, 202, 206, 218, 222, 232, 244, 260, 266, 268, 274, 290, 292, 298, 310, 326, 340, 344, 346, 362, 366, 372, 386, 394, 404, 412, 436, 466, 470, 474, 482, 490, 518, 520, 532, 534, 536, 546, 554, 562, 566, 580, 584, 596, 626, 634, 650, 652, 666, 680, 686, 688, 698, 706, 722, 724, 730, 732, 746, 772, 778, 786, 794, 808, 818, 834, 842, 850, 872, 874, 902, 906, 914, 922, 926, 932, 940, 962, 964, 974, 980, 986, 1018, 1036, 1038, ...

The first occurrence of k when σ (n) - n = k, and it is as follows: 2, 0, 4, 9, 0, 6, 8, 10, 15, 14, 21, 121, 27, 22, 16, 12, 39, 289, 65, 34, 18, 20, 57, 529, 95, 46, 69, 28, 115, 841, 32, 58, 45, 62, 93, 24, 155, 1369, 217, 44, 63, 30, 50, 82, 123, 52, 129, 2209, 75, 40, 141, 0, 235, 42, 36, 106, 99, 68, 265, 3481, 371, 118, 64, 56, 117, 54, 305, 4489, 427, 134, 201, 5041, 98, 70, 213, 48, 219, 66, 365, 6241, 147, 158, 237, 6889, 395, 166, 105, 0, 171, 78, 581, 88, 267, 116, 445, 0, 245, 9409, 1501, 124, 291, ...

The sequence of k's when σ (n) - n = k has no solutions (the Zeros above), and this is also the "untouchable" numbers of Paul Erdos. Wells, page 125. It is as follows: 2, 5, 52, 88, 96, 120, 124, 146, 162, 188, 206, 210, 216, 238, 246, 248, 262, 268, 276, 288, 290, 292, 304, 306, 322, 324, 326, 336, 342, 372, 406, 408, 426, 430, 448, 472, 474, 498, 516, 518, 520, 530, 540, 552, 556, 562, 576, 584, 612, 624, 626, 628, 658, 668, 670, 708, 714, 718, 726, 732, 738, 748, 750, 756, 766, 768, 782, 784, 792, 802, 804, 818, 836, 848, 852, 872, 892, 894, 896, 898, 902, 926, 934, 936, 964, 966, 976, 982, 996, 1002, 1028, 1044, 1046, 1060, 1068, 1074, 1078, 1080, 1102, 1116, 1128, ...

If a number is in parenthesis then that is not the value but the limit to which the test was run on my HP-71B. On the other hand, if the integer presented is Zero, then there is no possible answer. Often, a particular series maybe divided by two to save some room or to make clearer the sequence involved. The references for the above are in your bibliography as SI1, BE3 and AS1, plus "The Penguin Dictionary of Curious and Interesting Numbers," David Wells, Middlesex, England, 1986. If at some future date, I run across a filler or greatly extend the limits, I will forward the same to you.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:hp110+

Quotes:

"God invented 1,2 and 3, and man invented all the rest."
The author is unknown to me, but is a great quote for a book on numerical sequences. Maybe I'm thinking of the following quote.

"God himself made the whole numbers: everything else is the work of

man." Leoplod Kronecker
"The trouble with integers is that we have examined only the small ones." Ronald Graham
"The primary source of all mathematics are the integers." Herman Minkowski
"I am ill at these numbers." Wm. Shakespeare (Hamlet)

2

1 January 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Sir, this is a updated version of an earlier letter.

In the above referenced text, three sequences are cited (correctly, I might add) as simply "A Self-Generating Sequence." They are SSN (for Sloane Sequence Nbr. and not for Social Security Nbr.) 201, 231, and 909. In "Computers in Number Theory" edited by A.O.L. Atkin and B.J. Birch, Academic Press 1971, pages 249 to 257, (your reference is AT1) these three sequences are called "Ulam's Summation Sequences." U2({1,2}) is SSN 201, U2({2,3}) is SSN 231, and U2({1,3}) is SSN 909. However on page 256 a fourth sequence U3({1,2,3}) is mentioned but not cited by you. Therefore; please consider the following sequences for your upcoming second edition: U3({1,2,3}), U3({1,2,4}), U3({1,3,4}), U3({2,3,4}), U4({1,2,3,4}), U5({1,2,3,4,5}), U6({1,2,3,4,5,6}), U7({1,2,3,4,5,6,7}), U8({1,2,3,4,5,6,7,8}), U9({1,2,3,4,5,6,7,8,9}), U10({1,2,3,4,5,6,7 U11({1,2,3,4,5,6,7,8,9,10,11}) and U12({1,2,3,4,5,6,7,8,9,10,11,12})..

U3({1,2,3}): 1, 2, 3, 6, 9, 10, 11, 12, 28, 29, 30, 53, 56, 57, 80, 82, 104, 105, 107, 129, 130, 132, 154, 155, 157, 179, 180, 182, 204, 205, 207, 229, 230, 232, 254, 255, 257, 279, 280, 282, 304, ... (the nth term for n>5: 3*n= 25*n -45, 3*n+1= 25*n -43, and 3*n+2= 25*n -21, Muller's Theorem) ... .

U3({1,2,4}): 1, 2, 4, 7, 10, 12, 16, 17, 32, 36, 42, 57, 72, 73, 98, 102, 104, 129, 159, 164, 174, 189, 199, 221, 224, 255, 286, 287, 347, 372, 378, 403, 428, 443, 444, 469, 494, 529, 560, 586, 592, 652, 683, 718, 743, 780, 784, 865, 871, 957, 963, 988, 1085, 1110, 1114, 1155, 1176, 1206, 1236, 1293, 1302, 1460, 1516, 1541, 1633, 1639, 1791, 1827, 1852, 1882, 1918, 1978, 2009, 2035, 2070, 2126, ...

U3({1,3,4}): 1, 3, 4, 8, 12, 13, 15, 16, 18, 40, 42, 52, 77, 78, 79, 87,114, 116, 117, 152, 154, 188, 224, 228, 230, 260, 262, 263, 300, 301, 302, 336, 338, 375, 407, 409, 411, 412, 444, 446, 481, 482, 517, 521, 554, 555, 591, 626, 630, 631, 663, 665, 700, 701, 736, 740, 773, 774, 810, 845, 849, 850, 882, 884, 919, 920, 955, 959, 992, 993, 1029, ...(the nth term for n>8: 4*n+1=73*n-249 and if n=0 mod 3 then +3, 4*n+2= the previous term +1, 4*n+3= 73*n -213 and if n=2 mod 3 then +1, and 4*n+4= 73*n-177 and if n=0 mod 3 then +34 or if n=1 mod 3 then +32) ...

U3({2,3,4}): 2, 3, 4, 9, 14, 15, 16, 19, 23, 24, 55, 59, 60, 63, 64, 104, 105, 109, 112, 114, 155, 157, 159, 160, 203, 204, 206, 207, 208, 253, 254, 255, 258, 302, 305, 307, 308, 351, 352, 354, 355, 356, 401, 402, 403, 406, 450, 453, 455, 456, 499, 500, 502, 503, 504, 549, 550, 551, 554, 598, 601, 603, 604, 647, 648, 650, 651, 652, 697, 698, 699, 702, 746, 749, 751, 752, 795, 796, 798, ... (the nth term for n>1: 13*n= 148*n -92, 13*n+1= 148*n -90, 13*n+2= 148*n -89, 13*n+3=148*n -88, 13*n+4= 148*n -43, 13*n+5= 148*n -42, 13*n+6= 148*n -41, 13*n+7= 148*n -38, 13*n+8= 148*n +6, 13*n+9= 148*n +9, 13*n+10= 148*n +11, 13*n+11= 148*n +12, and 13*n+12= 148*n +55) ...

U4({1,2,3,4}): 1, 2, 3, 4, 10, 16, 17, 18, 19, 22, 64, 65, 66, 68, 69, 128, 132, 188, 190, 191, 194, 252, 253, 255, 313, 314, 318, 376, 377, 380, 438, 439, 498, 554, 556, 557, 560, 561, 620, 621, 680, 684, 740, 742, 743, 745, 803, 804, 863, 869, 923, 924, 925, 927, 928, 929, 988, 990, 1048, 1049, ...

U5({1,2,3,4,5}): 1, 2, 3, 4, 5, 15, 25, 26, 27, 28, 29, 35, 43, 45, 165,
  171, 172, 174, 180, 181, 328, 333, 338, 339, 340, 341, 493, 499, 500, 647,
  652, 657, 658, 659, 660, 661, 662, 663, 815, 818, 819, 971, 1127, (and no
  others less than 1130) ... .
U6({1,2,3,4,5,6}): 1, 2, 3, 4, 5, 6, 21, 36, 37, 38, 39, 40, 41, 51, 61,
  66, 284, 285, 289, 290, 297, 298, 299, 310, 312, 559, 561, 562, 570, 571,
  574, 575, 834, 835, 837, 838, 839, 840, 841, 849, 850, (and no others less
  than 1095) ... .
U7({1,2,3,4,5,6,7}): 1, 2, 3, 4, 5, 6, 7, 28, 49, 50, 51, 52, 53, 54, 55,
  70, 82, 91, 109, 112, 555, 556, 563, 564, 572, 573, 576, 583, 584, 591,
  593, (and no others less than 1053) ... .
U8({1,2,3,4,5,6,7,8}): 1, 2, 3, 4, 5, 6, 7, 8, 36, 64, 65, 66, 67, 68, 69,
  70, 71, 92, 106, 120, 141, 148, 792, 793, 802, 803, 816, 817, (and no others
  less than 818) ... .
U9({1,2,3,4,5,6,7,8,9}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 45, 81, 82, 83, 84,
  85, 86, 87, 88, 89, 117, 133, 153, 177, 189, 221, 225, (and no others less
  than 1139), ...
U10({1,2,3,4,5,6,7,8,9,10}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 55, 100, 101,
  102, 103, 104, 105, 106, 107, 108, 109, 145, 163, 190, 217, 235, 271, (and
  no others less than 278) ... .
U11({1,2,3,4,5,6,7,8,9,10,11}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 66, 121,
  122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 176, 196, 231, 261, 286,
  326, 341, 391, 396, (and no others less than 1849) ... .
U12({1,2,3,4,5,6,7,8,9,10,11,12}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,
  78, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 210, 232,
  276, 309, 342, 386, 408, 463, 474, (and no others less than 2473) ... .
for n>2, Un({1,2, ... ,n-1,n}): 1, 2, ... , n-1, n, °n(n+1), n^2 to n^2+n-1,
  (3*n^2-n)/2, (3*n^2+3*n-4)/2 for n>4, 2*n^2-n for n>4, 2*n^2+2*n-3 for n>6,
  (supposition only) ... .
These are all infinite series, provable along the lines employed by Euclid
  (Elements, Proposition 20, Book IX) to demonstrate the infinitude of the
  primes. Assume that X is the largest number in the sequence belonging
  to the Summation Series of n numbers at a time. Since these Xs are in numerical
  order, then X + X + ... + X + X represents a uniquely describable
  number. Therefore the assumption that X is that largest number in the
  sequence is false.

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF & GI
RGWv:hp110+
Rev:01-Supr22Sept92

31 August 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

In "Computers and the Imagination," Clifford A. Pickover, St. Martin's Press, Inc., New York, 1991, on pages 352-4, there is a "Grasshopper sequence" which grows from a binary tree. The sequence is as follows:

1, 4, 10, 12, 22, 26, 30, 46, 54, 62, 66, 78, 94, 110, 126, 134, 138, 158, 162, 186, 190, 222, 254, 270, 278, 282, 318, 326, 330, 374, 378, 382, 402, 446, 474, 510, 542, 558, 566, 570, 638, 654, 662, 666, 750, 758, 762, 766, 806, 810, 834, 894, 950, 954, 978, 1022, 1086, 1118, 1122, 1134, 1142, 1146, 1278, 1310, 1326, 1334, 1338, 1502, 1518, 1526, 1530, 1534, 1614, 1622, 1626, 1670, 1674, 1698, 1790, 1902, 1910, 1914, 1958, 1962, 1986, 2046, 2174, 2238, 2246, 2250, 2270, 2274, 2286, 2294, 2298, 2418, 2558, 2622, 2654, 2670, 2678, 2682, 2850, 3006, 3038, 3054, 3062, 3066, 3070, 3230, 3246, 3254, 3258, 3342, 3350, 3354, 3398, 3402, 3426, 3582, 3806, 3822, 3830, 3834, 3918, 3926, 3930, 3974, 3978, 4002, 4094, 4350, 4478, 4494, 4502, 4506, 4542, 4550, 4554, 4574, 4578, 4590, 4598, 4602, 4838, 4842, 4866, 5010, 5118, 5246, 5310, 5342, 5358, 5366, 5370, 5702, 5706, 5730, 5874, 6014, 6078, 6110, 6126, 6134, 6138, 6142, 6462, 6494, 6510, 6518, 6522, 6686, 6702, 6710, 6714, 6738, 6798, 6806, 6810, 6854, 6858, 6882, 7166, 7614, 7646, 7662, 7670, 7674, 7838, 7854, 7862, 7866, 7950, 7958, 7962, 8006, 8010, 8034, 8190, 8702, 8958, 8990, 9006, 9014, 9018, 9086, 9102, 9110, 9114, 9150, 9158, 9162, 9182, 9186, 9198, 9206, 9210, 9678, 9686, 9690, 9734, 9738, 9762, 10022 (the 225th term), ...

Sequentially yours,
Robert G. Wilson v,
Ph. D., ATP/CF & GI
RGWv:hp110+

11 September 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequence for inclusion in your forthcoming second edition of the above. Beginning op page 231, Mr. Pickover presents the "Juggler" problem.

The sequence is as follows: 1, 6, 2, 5, 2, 4, 2, 7, 7, 4, 7, 4, 7, 6, 3, 4, 3, 9, 3, 9, 3, 9, 3, 11, 6, 6, 6, 9, 6, 6, 6, 8, 6, 8, 3, 18, 3, 14, 3, 5, 3, 6, 3, 6, 3, 6, 3, 11, 5, 11, 5, 11, 5, 11, 5, 5, 5, 11, 5, 11, 5, 5, 3, 5, 3, 11, 3, 14, 3, 5, 3, 8, 3, 8, 3, 19, 3, 8, 3, 10, 8, 8, 8, 11, 8, 10, 8, 11, 8, 11, 8, 11, 8, 8, 8, 11, 8, 11, 8, 8, 8, ...

The alternate sequence is as follows: 1, 6, 2, 5, 2, 13, 7, 10, 7, 4, 7, 6, 3, 9, 3, 9, 3, 12, 3, 9, 6, 9, 6, 19, 6, 9, 6, 9, 6, 16, 3, 5, 3, 8, 3, 16, 3, 5, 3, 14, 3, 11, 14, 11, 14, 5, 14, 14, 14, 14, 14, 5, 14, 5, 14, 11, 8, 11, 8, 8, 8, 8, 8, 11, 8, 11, 8, 8, 8, 8, 8, 21, 11, 21, 11, 8, 11, 8, 11, 19, 11, 11, 11, 8, 11, 11, 11, 11, 11, 11, 8, 19, 8, 8, 8, 10, 8, 10, 8, ...

Reference: Clifford A. Pickover, Computers and the Imagination, St. Martin's Press, Inc., NY., 1991

Sequentially yours,
Robert G. Wilson v,
Ph. D., ATP/CF & GI
RGWv:hp110+

23 September 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

In the above referenced text, the most natural sequence is cited as simply "The Natural Numbers." It is SSN (for Sloane Sequence Nbr. and not for Social Security Nbr.) 173. There are fifteen elementary additional sequences that are all very simple. In fact, that is exactly what they are, the natural numbers, but not in the base Ten, but in bases two thru sixteen. Therefore; please consider the following sequences (all are to 101 terms for consistency) for your upcoming second edition:

Base 2: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111, 100000, 100001, 100010, 100011, 100100, 100101, 100110, 100111, 101000, 101001, 101010, 101011, 101100, 101101, 101110, 101111, 110000, 110001, 110010, 110011, 110100, 110101, 110110, 110111, 111000, 111001, 111010, 111011, 111100, 111101, 111110, 111111, 1000000, 1000001, 1000010, 1000011, 1000100, 1000101, 1000110, 1000111, 1001000, 1001001, 1001010, 1001011, 1001100, 1001101, 1001110, 1001111, 1010000, 1010001, 1010010, 1010011, 1010100, 1010101, 1010110, 1010111, 1011000, 1011001, 1011010, 1011011, 1011100, 1011101, 1011110, 1011111, 1100000, 1100001, 1100010, 1100011, 1100100, 1100101, ...

Base 3: 1, 2, 10, 11, 12, 20, 21, 22, 100, 101, 102, 110, 111, 112, 120, 121, 122, 200, 201, 202, 210, 211, 212, 220, 221, 222, 1000, 1001, 1002, 1010, 1011, 1012, 1020, 1021, 1022, 1100, 1101, 1102, 1110, 1111, 1112, 1120, 1121, 1122, 1200, 1201, 1202, 1210, 1211, 1212, 1220, 1221, 1222, 2000, 2001, 2002, 2010, 2011, 2012, 2020, 2021, 2022, 2100, 2101, 2102, 2110, 2111, 2112, 2120, 2121, 2122, 2200, 2201, 2202, 2210, 2211, 2212, 2220, 2221, 2222, 10000, 10001, 10002, 10010, 10011, 10012, 10020, 10021, 10022, 10100, 10101, 10102, 10110, 10111, 10112, 10120, 10121, 10122, 10200, 10201, 10202, ...

Base 4: 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33, 100, 101, 102, 103, 110, 111, 112, 113, 120, 121, 122, 123, 130, 131, 132, 133, 200, 201, 202, 203, 210, 211, 212, 213, 220, 221, 222, 223, 230, 231, 232, 233, 300, 301, 302, 303, 310, 311, 312, 313, 320, 321, 322, 323, 330, 331, 332, 333, 1000, 1001, 1002, 1003, 1010, 1011, 1012, 1013, 1020, 1021, 1022, 1023, 1030, 1031, 1032, 1033, 1100, 1101, 1102, 1103, 1110, 1111, 1112, 1113, 1120, 1121, 1122, 1123, 1130, 1131, 1132, 1133, 1200, 1201, 1202, 1203, 1210, 1211, ...

Base 5: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 100, 101, 102, 103, 104, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 142, 143, 144, 200, 201, 202, 203, 204, 210, 211, 212, 213, 214, 220, 221, 222, 223, 224, 230, 231, 232, 233, 234, 240, 241, 242, 243, 244, 300, 301, 302, 303, 304, 310, 311, 312, 313, 314, 320, 321, 322, 323, 324, 330, 331, 332, 333, 334, 340, 341, 342, 343, 344, 400, 401, ...

Base 6: 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 55, 100, 101, 102, 103, 104, 105, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 200, 201, 202, 203, 204, 205, 210, 211, 212, 213, 214, 215, 220, 221, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, ...

Base 7: 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 160, 161, 162, 163, 164, 165, 166, 200, 201, 202, 203, ...

Base 8: 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, ...

Base 9: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, ...

Base 10: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, ...

Base A: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 8A, 90, 91, 92, ...

Base B: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, 7B, 80, 81, 82, 83, 84, 85, ...

Base C: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 6C, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 7A, ...

Base D: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 3D, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 4D, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 5D, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 6A, 6B, 6C, 6D, 70, 71, 72, 73, ...

Base E: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 2E, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 3A, 3B, 3C, 3D, 3E, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A, 4B, 4C, 4D, 4E,

50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C, 5D, 5E, 60, 61, 62,
63, 64, 65, 66, 67, 68, 69, 6A, 6B, ...
Base F: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14,
15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26,
27, 28, 29, 2A, 2B, 2C, 2D, 2E, 2F, 30, 31, 32, 33, 34, 35, 36, 37, 38,
39, 3A, 3B, 3C, 3D, 3E, 3F, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 4A,
4B, 4C, 4D, 4E, 4F, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 5A, 5B, 5C,
5D, 5E, 5F, 60, 61, 62, 63, 64, 65, ...

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

23 September 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
On a new Iterated Prime Number Series
Dear Dr. Sloane,

In the above referenced text, the Primes and recurrences occuppy a large portion of the index.

Although iteration and its brother recursion dates back several centuries, in this last decade it has enjoyed equal footing with other numerical methods, I believe I can safety say that this phenomenon is due for the most part to the current interest in "chaos." Being in vogue has brought many other fringe elements into the forefront.

Many processes use iteration as a technique for problem solving. Others are employed for their own sake, as a mathematical amusement. In the former class we have the famous Newton-Raphson method of root extraction (this is not to be confused with the dental proceedure). An example of the later is the famous 3*X +1 or - 2 problem. In the Newton method you have a process which you can observe the speed of convergence, in this case quadratically, to the true answer. In the later the number of iterations it takes for an integer to cascade to one wildly oscillates and today is still an unsolved number theory problem.

The series I wish to present is, I'm sure, of the later, although its progression is much better defined than the Collatz function. The audience is well acquianted with (x), hereinafter denoted as pi(x), which is the arithematic function that counts the number of primes less than or equal to the integer x, with the number one being defined as neither a prime nor a composite number. As an example, the pi(10)=4, they being the primes 2, 3, 5 & 7. There is that noteworthy theorem which states that the Limit of pi(x) as x approaches infinity is the quotient of x divided by the natural logarithm of x. Also by the definition itself, it is obvious that x always exceeds the pi(x). Unlike the hailstone problem though, we know that taking succesive iterations of the pi(x) -> x and counting the number of iterations, we always know that this prodeedure will terminate at Zero. Lets call this new function the Q(n) and define it as the number of steps (iterations) we must perform the pi(x) before pi(x)=0. Examples, the Q(10)=4 because pi(10)->4, pi(4)->2, pi(2)->1 and pi(1)->0. We performed the operation pi() four times.

Now let us investigate the growth of Q(n). Since the plot of the pi(x) is relatively smooth, should we not expect the Q(n) to behave similarly? Second, at what values does the Q(n) increment from a lower value to a higher value? The seccesive values of n, the Q(n) will stay the same or increase by just one; it can never go down. The reader is left with his own proof. It is similar to the proof that of pi(x) behaves in the similar manner.

In fact, it can be shown that the first time that Q(n)= whatever, can be generated by the inverse function of pi(x), that is P , i being the index. Therefore, we have the following obviously incomplete table:

X		P	number
1	0	1	
2	1	2	

3	2	3
4	3	5
5	5	11
6	11	31
7	31	127
8	127	709
9	709	5,381
10	5,381	52,711
11	52,711	648,391
12	648,391	9,737,333
13	9,737,333	174,440,041
14	174,440,041	3,657,500,101
15	3,657,500,101	88,362,852,307
16	88,362,852,307	2,428,096,940,717
17	2,428,096,940,717	~ 74,900,000,000,000
18	< 74,900,000,000,000	~ 2,560,000,000,000,000

references:

M. Abramowitz and I. Stegun, "Hdbk of Math Fnctns." p870-873
Solomon W. Golomb, E 3385 [1990,427] The Am. Math. Mo. v98n9p858-9, Nov 91.
D. Knuth, "The Art of Computer Programming," v2p365
Hans Riesel, "Prime Numbers and Computer Methods for Factorization"
Stephen Wolfram "Mathmatica," the function Prime[]

Sequentially yours;
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:hp110+

19 January 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following sequences for inclusion in your forth-coming second edition of the above. "A positive integer is called 'Powerful' if and only if it can be expressed as a sum of positive integral powers of its digits." A favorite example of a dear friend of mine is 153 = 1^3 + 5^3 + 3^3. "Clearly, the positive integers from one to nine are trivially powerful, since each can be written as its own first power."
The sequence is as follows: (1, 2, 3, 4, 5, 6, 7, 8, 9) 24, 43, 63, 89, 132, 135, 153, 175, 209, 224, 226, 262, 264, 267, 283, 332, 333, 334, 357, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 407, 445, 463, 518, 598, 629, 739, 794, 849, 935, 994, 1034, 1073, 1074, 1234, 1255, 1306, 1323, 1326, 1349, ... For a further rendition, consult the attached pages for the first 3464 terms which are all the "Powerful" numbers less than 100,000. The number of terms per ten thousand is the following: 476, 144, 546, 597, 306, 258, 468, 242, 207, 220, 15, 49, 225, ...
Some of the above are represented by two or more uniquely different set of exponents, some are primes, some are sequential for a time, some are represented by exponents equal to the some combination of digits of the integer, some are represented by contiguous exponents, and some are repre-sented by exponents equal to the number of digits of the integer. Each of these are subsets of the above and represent all of the members (the count is in parenthesis after the last term) that are less than 100,000.
The first of these six sequences (two or more uniquely different set of exponents) is as follows: 264, 373, 375, 935,
2139, 2203, 2223, 2243, 2245, 2263, 2332, 2334, 2336, 2352, 2356, 2372, 2376, 2394, 2407, 2427, 2536, 2537, 2664, 2733, 2793, 2824, 2843, 2932,
2933, 2939, 3257, 3292, 3298, 3324, 3342, 3435, 3437, 3457, 3477, 4132, 4224, 4225, 4228, 4243, 4245, 4249, 4262, 4265, 4288, 4332, 4352, 4353, 4355, 4397, 4572, 4573, 4732, 4733, 4933, 4959, 5323, 5325, 5432, 5832, 6424, 6793, 6934, 6939, 7329, 7639, 7906, 8208, 8224, 8225, 8262, 8283, 8289, 8376, 8396, 8825, 9273, 9274, 9324, 9342, 9723,
12324, 12347, 13933, 14363, 16293, 16363, 16432, 16434, 16472, 16479, 16524, 16743, 16927, 17243, 17247, 17248, 17249, 17848, 19243, 19732, 19733,
20933, 21258, 22243, 22247, 22249, 22283, 22285, 22338, 22373, 22375, 22395, 22465, 22489, 22539, 22593, 22628, 22732, 22733, 22738, 22848, 22864, 22883, 22935, 22953, 22954, 22955, 22958, 22995, 23235, 23237, 23253, 23275, 23295, 23297, 23345, 23387, 23417, 23433, 23437, 23472, 23473, 23474, 23476, 23492, 23493, 23497, 23498, 23543, 23548, 23562, 23582, 23584, 23634, 23652, 23697, 23723, 23749, 23835, 23852, 23856, 23875, 23897, 23924, 23928, 23943, 23945, 23947, 23949, 23967, 23988, 24137, 24226, 24229, 24265, 24289, 24375, 24376, 24379, 24396, 24397, 24483, 24489, 24532, 24536, 24593, 24622, 24623, 24627, 24628, 24644, 24647, 24664, 24667, 24680-9, 24736, 24756, 24759, 24796, 24823, 24843, 24845, 24848, 24864, 24865, 24868, 24883, 24953, 24976,
25235, 25254, 25257, 25273, 25432, 25437, 25439, 25453, 25473, 25476, 25479, 26224, 26243, 26246, 26247, 26339, 26352, 26393, 26537, 26593, 26713, 26739, 26793, 27234, 27322, 27346, 27363, 27367, 27369, 27473, 27568, 27923, 27984, 28264, 28265, 28395, 28425, 28533, 29637, 29967,
30349, 32235, 32252, 32275, 32297, 32436, 32524, 32525, 32749, 32786, 32812,

32813, 32814, 32816, 32830-9, 32852, 32854, 32857, 32859, 32872, 32873, 32874, 32877, 32890-9, 32907, 32908, 32922, 32924, 32926, 32942, 32943, 32945, 32948, 32949, 32968, 32969, 32985, 32987, 32989, 33022, 33028, 33042, 33225, 33227, 33228, 33229, 33246, 33262, 33263, 33266, 33268, 33284, 33398, 33427, 33428, 33429, 33442, 33486, 33539, 33592, 33752, 33798, 33804, 33822, 33828, 33829, 33842, 33844, 33847, 33882, 34234, 34236, 34238, 34239, 34256, 34272, 34292, 34297, 34298, 34299, 34322, 34326, 34474, 34525, 34727, 34728, 34746, 34762, 34872, 34892, 34922, 34924, 34982, 35223, 35224, 35225, 35245, 35247, 35248, 35284, 35287, 35352, 35625, 35628, 35932, 35938, 35939, 35992, 36292, 36324, 36342, 36382, 36385, 36472, 36722, 36723, 36724, 36922, 36984, 37223, 37224, 37226, 37228, 37229, 37242, 37243, 37244, 37248, 37264, 37284, 37352, 37372, 37424, 37426, 37428, 37442, 37482, 37648, 38322, 38522, 39338, 39352, 39372, 39392, 39393, 39402, 39422, 39423, 39427, 39428, 39429, 39442, 39446, 39462, 39482, 39483, 39538, 39624, 39628, 39642, 39662, 39682, 39687, 39752, 39758, 39823, 39844, 39882, 39886,

40629, 40689, 42243, 42248, 42263, 42264, 42268, 42539, 42624, 42689, 42759, 43498, 43836, 43926, 44648, 46712, 46716, 46730-9, 46753, 46792, 46794, 46796, 46824, 46825, 46932, 46933, 46935, 46953, 46973, 46975, 46976, 47236, 47363, 47653, 47693, 47726, 48625, 49323, 49436, 49637, 49722, 49725, 49728, 49876,

52459, 52542, 52639, 52748, 52789, 52857, 53249, 53262, 53266, 53282, 53486, 53628, 53862, 53956, 54259, 54363, 55426, 56258, 56433, 56439, 56632, 56639, 59192, 59193, 59222, 59223, 59226, 59242, 59246, 59247, 59263, 59267, 59269, 59283, 59312, 59314, 59332, 59333, 59335, 59339, 59356, 59357, 59392, 59394, 59423, 59442, 59443, 59462, 59463, 59683, 59734, 59822, 59823, 59863, 59883, 59912, 59913, 59938, 59992, 59993, 60352, ...

The sequence with three uniquely different set of exponents is as follows: 2203, 2332, 4224, 4228, 4245, 8224, 13933, 16432, 19733, 22465, 22489, 22953, 22954, 23235, 23237, 23275, 23437, 23472, 23473, 23492, 23584, 23835, 23924, 24265, 24289, 24376, 24623, 24647, 24683, 24684, 24687, 24843,

25254, 25479, 26243, 26339, 26793, 27363, 28265, 28425, 32813, 32830, 32831, 32832, 32835, 32838, 32839, 32857, 32873, 32877, 32894, 32949, 32968, 32987, 33262, 33268, 33284, 33592, 33752, 33842, 34326, 34872, 35932, 37223, 37228, 37442, 38322, 39422, 39483, 39538, 42243, 42624, 46732, 46733, 46825, 46935, 46973, 49728, 52857, 53862, 59247, 59267, 59332, 59392, 59423, 59463, 59822, 59913, ...

The sequence with four uniquely different set of exponents is as follows: 2334, 2537, 4243, 22243, 22247, 22249, 23493, 23497, 23875, 23897, 23943, 24229, 24628, 24736, 24756, 24845, 24848, 26247, 28264, 32816, 32833, 32834, 32836, 32943, 33227, 34239, 36324, 36724, 37226, 37242, 37428, 39429, 39442, 39844, 46932, 59222, 59993, ...

The sequence with five uniquely different set of exponents is as follows: 25432, 26224, 32524, 32812, 32874, 32922, 32924, 32948, 32985, 33829, 39423, 39428, 39642, 42264, 53282, ... ? ..., 122274, ...

The sequence with six uniquely different set of exponents is as follows: 22864, 24823, 24883, 32837, 32892, 32893, 32896, 39482, 59223, 59823, ...

The sequence with seven uniquely different set of exponents is as follows: 23852, 35224, ...

The sequence that follows are those members occurring first of the original set (see the first term of the previous seven sequences) which can be represented by n uniquely different set of exponents: 24, 264, 2203, 2334, 25432, 22864, 23852, ...

The sequence that follows are those members occurring first of the full decade set (see only ten "Powerful" numbers on a subsequent page) which can be represented by n uniquely different set of exponents: 370, 24680,

32830, ...
The second of these six sequences (the primes) is as follows: 43, 89, 283, 373, 379, 463, 739, 2063, 2083, 2131, 2137, 2179, 2203, 2243, 2333, 2423, 2753, 2803, 2843, 2939, 2953, 3167, 3257, 3457, 3527, 3943, 3947, 4133, 4153, 4157, 4159, 4241, 4243, 4261, 4339, 4373, 4397, 4447, 4649, 4733, 4933, 4951, 4957, 5323, 5653, 5657, 6247, 6793, 7639, 8209, 8243, 8263, 8443, 8623, 8647, 8971, 9697,
10723, 12343, 12347, 13337, 13933, 14369, 16363, 16433, 16453, 16477, 16729, 16927, 17443, 17483, 17737, 18617, 20483, 20887, 21323, 22247, 22283, 22447, 22573, 22643, 22973, 23053, 23297, 23327, 23417, 23473, 23497, 23549, 23563, 23743, 23747, 23857, 24043, 24137, 24229, 24337, 24373, 24379, 24593, 24623, 24683, 24733, 24841, 24847, 24953, 24977, 25127, 25237, 25253, 25367, 25439, 25453, 25657, 26153, 26249, 26339, 26393, 26647, 26713, 27253, 27361, 27367, 27479, 27527, 27617, 27697, 27749, 28403, 28513, 28603, 29347, 29947, 30341, 30347, 32297, 32327, 32587, 32749, 32783, 32789, 32831, 32833, 32839, 32941, 32969, 32983, 32987, 33023, 33029, 33223, 33247, 33427, 33623, 33757, 33829, 34217, 34231, 34297, 34327, 34367, 34439, 34981, 35081, 35083, 35089, 35221, 35227, 35447, 35809, 35933, 36277, 36343, 36457, 36473, 36497, 36637, 36877, 37223, 37243, 37423, 37483, 37663, 37997, 38567, 39373, 39443, 39623, 39827, 39869, 42407, 42683, 42689, 42863, 42953, 44449, 44641, 44647, 44683, 46771, 46933, 46993, 47363, 47563, 47639, 47653, 47699, 49727, 49891, 52583, 52639, 52709, 52783, 52837, 54727, 55843, 56633, 59063, 59069, 59243, 59263, 59333, 59357, 59443, 59467, 59791, 59797, 59863, 59957, 60289, 60337, 60353, 60397, 60737, 61673, 62539, 62753, 63149, 63299, 63473, 63493, 63527, 63697, 63761, 64283, 64333, 64663, 64793, 64937, 65839, 66359, 66863, 67343, 67433, 67493, 68449, 68669, 68777, 69473, 72497, 73643, 74293, 74323, 74363, 74923, 75937, 75991, 75997, 76079, 76369, 76943, 77849, 78541, 78653, 79283, 79843, 80537, 81293, 82241, 82373, 82487, 83257, 83341, 85037, 85237, 85439, 85639, 86243, 86249, 86353, 87253, 89417, 91823, 92383, 92387, 92723, 93283, 93427, 93559, 94529, 94543, 94727, 95287, 95737, 96493, 98347, 98389, 98473, 98729, 98893, 98897 (312), 106219, 112643, 117673, 117701, 117703, 117709, 117763, 118973, 119237, 119747, 122323, 122347, 124633, 124673, 124679, 124769, 124987, 125243, 125683, 125687, 125863, 126457, 126493, 126839, 127373, 127867, 128327, 128347, 129443, 131149, 131321, 132049, 132173, 132263, ...
The third of these six sequences are a finite number of consecutive (siblings?) integers.
The sequence, which I will term "twins" after the example of the primes, consists with only two "Powerful" numbers in a row, is as follows: 1073, 2263, 2314, 2352, 2355, 2372, 2464, 2536, 2863, 3214, 3254, 3294, 3453, 3725, 3764, 4132, 4224, 4352, 4396, 4648, 4732, 4735, 4932, 5258, 5366, 5832, 6423, 6934, 8262, 8288, 9236, 10693, 10722, 13132, 14362, 15422,
16742, 17463, 19732, 20355, 20932, 21235, 21348, 22337, 22557, 22732, 22972, 22994, 23274, 23347, 23364, 23476, 23492, 23522, 23548, 23562, 23763, 23852, 23924, 24264, 24336, 24352, 24372, 24375, 24396, 24643, 24936, 24976, 25253, 25272, 25275, 25322, 25366, 25476, 26393, 26426, 27472, 27497, 28532, 28553, 29346, 29435, 32436, 32524, 32527, 32587, 32907, 32926, 32968, 33246, 33283, 33538, 33685, 33778, 33828, 33846, 33864, 34322, 34326, 34366, 34387, 34525, 34528, 34727, 34947, 35247, 35284, 35487, 35978, 36253, 36274, 36342, 36437, 36567, 36656, 36742, 37267, 37352, 37392, 37442, 37826, 38348, 38382, 38566, 38834, 39372, 39392, 39442, 39532, 39623, 42598, 42623, 42863, 43473, 43527, 46372, 46682, 46862, 46993, 47362, 47367, 47652, 47967, 49385, 49784, 52522, 53262, 53862, 54328, 54632, 56858, 57262, 59138, 59192, 59202, 59226, 59282, 59332, 59335, 59356, 59442, 59462, 59912, 59992, 61322, 62357, 62624, 62752, 63342, 63492, 63497, 63946, 64572, 65693, 65838, 66392, 66842, 66953, 67292, 67742, 68224, 68753, 68824, 68958, 69236, 69253, 69273, 69434, 69472, 72234, 73464, 74345, 74362, 74436, 74632, 76236, 76382, 76743, 78253, 78523, 78526,

79264, 79682, 79758, 80536, 82357, 82553, 82556, 82774, 85272, 85293, 85432, 85724, 87634, 88352, 91842, 91862, 92324, 92342, 92387, 92522, 92582, 92677, 93868, 94236, 94542, 94562, 94652, 95286, 95462, 96542, 98388, 98728, 99682 (234), 117672, 118172, 122346, 122436, 122782, 123578, 124922, 124946, 124962, 124966, 126326, 126363, 126673, 126762, 126766, 127265, 127685, 129442, 129758, 132227, 132263, ...

The sequence, which I will term "triples" in keeping with the previous, with only three "Powerful" numbers in a row is as follows: 332, 2062, 2223, 2394, 2626, 2737, 2792, 2932, 3435, 4572, 4622, 5342, 6792, 8223, 9272, 23235, 23472, 23496, 23582, 23743, 23747, 23856, 24735, 24863, 25235, 27322, 27345, 32325, 32872, 32922, 33427, 33622, 33822, 33842, 34872, 36472, 36722, 37242, 38324, 39627, 46823, 49236, 49322, 52832, 56632, 59222, 59822, 62352, 63122, 63542, 64463, 66937, 67342, 67432, 69727, 73642, 74232, 74322, 76232, 76323, 76452, 82263, 85324, 85342 (64), 126345, 127865, 132172, 132242, ...

The "quadruples" sequence with only four "Powerful" numbers in a row is as follows: 3474, 22332, 23324, 37262, 39482, 52723, 62394, 63722, 64332, 64792, 67322 (11), ...

The "quituples" sequence with only five "Powerful" numbers in a row is as follows: 2332, 3234, 22372, 22535, 23435, 37482 (6), ...

The "septuples" sequence with only seven "Powerful" numbers in a row is as follows: 63432 (1), ... There are no "sextuples!"

The "dectuples" sequence with only ten "Powerful" numbers, a full decade (in a row having as its first member a number congruent to zero modulus ten) is as follows: 370, 2130, 2240, 3940, 4150, 4240, 4260, 4950, 5320, 8200, 8970, 16430, 16470, 17240, 17840, 22950, 24590, 24620, 24680, 24750, 24840, 25430, 26240, 27360, 28260, 28730, 30340, 32780, 32810, 32830, 32890, 32940, 32980, 33020, 33220, 33260, 34230, 34290, 34980, 35080, 35220, 35930, 36980, 37220, 37420, 39420, 42240, 42260, 44640, 46710, 46730, 46770, 46790, 46930, 46970, 47690, 49720, 49890, 52700, 53480, 59060, 59240, 59790, 59930, 60350, 62640, 62660, 63290, 63320, 63470, 63760, 64730, 66350, 66480, 68640, 75990, 78540, 78650, 79590, 82240, 82370, 82440, 83340, 84950, 85230, 91820, 93420, 94520, 95240, 98340, 98890 (91), 117700, 124630, 127750, 131320, ... This list could be presented shorter by dividing all entries by ten.

And that's it for the "tuples!" The first occurrence of "n-tuples" is: 1073, 332, 3474, 2332, xxxxx, 63432, xxxxx, xxxxx, 370, xxxxx, ...

This sequence consists of decades (plus the number of terms - cousins?) which have more than two integers in it but less then ten integers and not included singularly in any of the previous set of five sequences immediately proceeding this one: 263, 2063, 2224, 2354, 2374, 2443, 2533, 2573, 2624, 2735, 2934, 3254, 3294, 3434, 3453, 3523, 3723, 4133, 4223, 4443, 4643, 4735, 5363, 6243, 6935, 6973, 8224, 8283, 9233, 12343, 14363, 17333, 17373, 17443, 19733, 20353, 20533, 21343, 22243, 22263, 22336, 22376, 22463, 22536, 22593, 22734, 22973, 23235, 23273, 23293, 23344, 23436, 23475, 23495, 23524, 23543, 23563, 23584, 23746, 23855, 23923, 23944, 24334, 24375, 24483, 24643, 24663, 24735, 24864, 24933, 25234, 25253, 25274, 25323, 25474, 25653, 26333, 27324, 27345, 27473, 27493, 28533, 29344, <(30000), ...

The group of series that follow measure the distance or "gap" between neighbors.

This sequence consists of two numbers seperated by one; that is a difference of two: 224, 262, 2203, 2225, 2353, 2443, 2572, 2733, 2735, 3145, 3165, 3212, 3252, 3255, 3292, 3523, 3525, 3723, 4353, 4403, 4447, 4733, 4736, 5364, 6247, 6932, 6935, 6937, 6972, 6974, 7962, 8243, 9234, 9723, 12343, 12345, 12432, 17332, 17376, 17443, 22247, 22267, 22283, 22335, 22376, 22393, 22463, 22465, 22533, 22593, 22595, 22733, 22753, 22823, 23034, 23162, 23237, 23272, 23295, 23343, 23345, 23433, 23452, 23474, 23527, 23745, 23835, 23943, 23945, 23947, 24332, 24334, 24373, 24487, 24667, 24733, 24737, 24994, 25273, 25477, 25657, 26337, 26352, 27324, 27343, 27347, 27747, 28423, 29347, 32525, 32635, 32852, 32857, 32903, 32905, 32924, 32963, 33284, 33286, 33824, 33826, 33844, 33882, 34036, 34254, 34272, 34472, 34526, 34672, 34784, 34832, 34834, 34922, 35022, 35203, 35243, 35245, 35285, 35287, 35885, 35992, 36232, 36237, 36275, 36292, 36294, 36346, 36435, 36746, 36922, 37244, 37246, 37265, 38322, 38343, 39352, 39536, 39687, 39823, 39825, 39882, 39884, 42534, 42687, 42864, 43236, 43432, 43523, 43525, 43542, 43782, 43924, 47944, 49234, 49272, 49274, 52362, 52743, 53222, 53224, 53226, 53647, 54326, 59224, 59267, 59312, 59333, 59373, 59392, 59734, 59736, 60593, 62532, 62733, 63232, 63362, 63493, 63495, 63527, 63547, 63695, 63704, 63742, 63744, 63942, 63944, 63985, 64335, 65232, 65322, 65722, 65877, 66423, 66443, 66532, 66534, 66732, 66993, 67232, 67234, 67293, 67325, 67327, 67344, 67346, 67727, 67896, 67923, 67942, 67982, 68243, 68423, 69473, 69922, 72232, 72362, 72833, 74234, 74296, 74923, 75352, 76234, 76237, 76294, 76362, 76927, 78273, 78295, 78524, 79203, 79332, 79534, 79756, 82355, 82554, 82572, 82643, 82685, 82953, 83252, 83672, 83674, 84755, 84757, 85433, 87632, 92895, 92922, 94297, 94675, 94725, 95795, 96744, 98322 (254), 117722, 117724, 120722, 121752, 123537, 124787, 126297, 126327, 126343, 126387, 126477, 126523, 127283, 127332, 127334, 127373, 127592, 131342, 131362, 132225, 132244, 132264, ...

This sequence consists of two numbers seperated by two: 132, 264, 1323, 2373, 2376, 2574, 2623, 3295, 3342, 3432, 3454, 3566, 3746, 4133, 4225, 4285, 6244, 7296, 8225, 17445, 19733, 20352, 20536, 21345, 21436, 22355, 22735, 22932, 23053, 23232, 23292, 23493, 23584, 23634, 23853, 23925, 24226, 24376, 24423, 24464, 24644, 24664, 24773, 24865, 25232, 25254, 25473, 26736, 27234, 27253, 27433, 28533, 29343, 29634, 32562, 32854, 32874, 32965, 33084, 33772, 33775, 33954, 34323, 34674, 35205, 35625, 35845, 36234, 36324, 36343, 36382, 36454, 36544, 36743, 37642, 37645, 38345, 38563, 39443, 39533, 39624, 42536, 42595, 48664, 49295, 52523, 52745, 52834, 53246, 53263, 56273, 56294, 59283, 59336, 59353, 59423, 59799, 62335, 62354, 62463, 62466, 62625, 62753, 62756, 62794, 62932, 63343, 63412, 63544, 63692, 64934, 66425, 66734, 66884, 66934, 67944, 69274, 73423, 73664, 74293, 74346, 75934, 76296, 76924, 79932, 82045, 82352, 82465, 82624, 82645, 83254, 85326, 85926, 92523, 93265, 95734, 96564 (137), 122476, 122783, 124963, 126273, 126474, 126763, 126836, 127486, 127682, 131232, 132064, 132266, ...

This sequence consists of two numbers seperated by three: 1672, 2423, 2445, 2532, 2753, 3492, 3674, 4443, 4644, 5232, 5653, 5873, 9474, 9693, 14722, 17334, 17372, 20532, 22243, 22263, 22624, 23523, 23723, 23963, 24483, 24532, 24572, 24712, 24792, 24932, 25323, 25342, 25653, 26333, 26533, 27493, 27635, 28332, 32252, 32899, 33242, 34762, 34924, 36254, 36474, 37332, 39844, 40685, 42624, 42683, 43232, 43434, 43474, 43582, 43674, 46805, 47363, 49364, 49872, 49922, 52364, 54655, 56382, 59263, 59463, 59602, 59772, 62425, 62735, 63498, 63502, 63675, 63832, 64445, 64593, 65672, 66374, 67925, 69293, 72385, 73465, 74324, 79572, 80352, 85344, 85435, 85963, 86373, 86554, 92383, 92454, 92723, 93663, 94474, 95282, 96272, 96522, 98542, 98922 (99), 122274, 125352, 125643, 125683, 126722, 131235, 131253, ...

This sequence consists of two numbers seperated by four: 2934, 3652, 5562, 8283, 9342, 13332, 17352, 20372, 22973, 23543, 23563, 23652, 23892, 25384, 25672, 25764, 26443, 33422, 33442, 33752, 34274, 36412, 36492, 37353, 38522, 39682, 39864, 40624, 47962, 52763, 56634, 62534, 62594, 63522, 63632, 66044, 67634, 73644, 74498, 76364, 85472, 92162, 92343, 98723 (44), 128942, ...

This sequence consists of two numbers seperated by five: 2843, 4397, 7323, 14363, 23673, 27473, 27562, 29253, 32743, 33532, 33792, 33798, 34562, 35352, 35372, 37443, 37732, 39642, 39752, 43492, 46799, 46953, 47673, 52783, 54363, 54742, 56433, 59596, 64283, 66843, 67522, 67543, 67898, 69343, 72343, 74363,

74492, 76432, 78922, 85273, 86243, 92362 (42), 124473, 124673, 124723, 124763, 124833, 125223, 125733, ...
This sequence consists of two numbers seperated by six: 4332, 7632, 24796, 39462, 52432, 52693, 63697, 69742, 75222, 79352 (10), 117693, 131142, ...
This sequence consists of two numbers seperated by seven: 24599, 28395 (2), 127594, ...
This sequence consists of two numbers seperated by eight: 2194, 2794, 39393, 42599, 59193 (5), ...
This sequence consists of two numbers seperated by nine: 2597, 23897, 28593, 47296, 52294, 53396, 62599, 65694, 69297, 98493 (10), 131092, ...
This sequence consists of two numbers seperated by ten: 2396, 4249, 24669, 24739, 27349, 32819, 32969, 34279, 42249, 46719, 46779, 46959, 47679, 53469, 62649, 65294 (16), ... This sequence seems to be the only one which has a pattern - they are mostly (87°%) conguent 9 mod 10.
Here is the series which represents the "gap" (see the first term in each of the above) of increasing magnitude:
332, 224, 132, 1672, 2934, 2843, 4332, 24599, 2194, 2597, 2396, 4228, 357, 2048, 209, 267, 1306, 135, 24, 43, 1234, 153, 334, 6579, 4574, 63, 3496, 379, 17378, 16494, 598, 25494, 6939, 175, 9237, 226, 19736, 407, 1034, 994, 25389, 16942, 89, 1498, 3167, 6603, 3385, 5384, 283, 5182, 1255, 22395, 12562, 26593, 463, 6976, 43926, 21439, 935, 6732, 12434, 49436, 4869, 26249, 5567, 5766, 16363, 2864, 2664, 3786, 1765, 20572, 38383, 8462, 20752, 42407, 27052, 62976, xxxxxx, 518, 99402, 78963, 6849, 3856, 66647, 849, 4485, 4738, 1676, 54476, 7032, 1542, 69634, 5453, 16634, 122673, 125977, 58962, xxxxxx, xxxxxx, 25942, 43983, 17737, 30236, 27129, 20377, 17026, 20644, 56749, 629, 7123, 1386, 28132, 16743, 57263, 98926, 82955, xxxxxx, 99483, 79083, ... The largest observed difference or "Gap" is 3992 (108234 to 112226).
The increasing Gap sequence, analogous to SSN 327, is as follows: 1, 9, 24, 43, 63, 89, 283, 463, 518, 629, 1074, 1836, 4959, 5902, 9963, 10322, 10723, 89497, 99843, 108234, ... And the Gap interval from the previous is as follows: 1, 15, 19, 20, 26, 43, 49, 55, 80, 110, 160, 212, 223, 342, 359, 371, 1575, 2323, 2611, 3992, ...
The sequences of neighboring Powerful numbers in arithmatic progression follows.
Only three in a row and in brackets the difference if other than one.
332, 739 [55], 2062, 2223, 2373 [3], 2394, 2626, 2733 [2], 2737, 2792, 2932, 3435, 3523 [2], 3678 [14], 4194 [15], 4405 [19], 4572, 4622, 5342, 6792, 6935 [2], 6972 [2], 8223, 9272, 12343 [2], 16942 [42], 22463 [2], 22593 [2],
23235, 23343 [2], 23472, 23496, 23582, 23743, 23747, 23856, 23858 [17], 24332 [2], 24735, 24863, 25235, 26555 [19], 27322, 27345, 28355 [20], 32325, 32872, 32903 [2], 32922, 33284 [2], 33427, 33622, 33772 [3], 33792 [6], 33822, 33824 [2], 33842, 34832 [2], 34872, 35243 [2], 35285 [2], 36292 [2], 36472, 36722, 37242, 37244 [2], 37642 [3], 38324, 39627, 39823 [2], 39882 [2], 43523 [2], 46823, 48226 [19], 49236, 49272 [2], 49322, 52832, 52837 [20], 53356 [20], 56632, 59222, 59734 [2], 59822, 62352, 62463 [3], 62753 [3], 63122, 63234 [20], 63493 [2], 63498 [4], 63542, 63742 [2], 63942 [2], 64463, 65258 [18], 66532 [2], 66937, 67232 [2], 67325 [2], 67342, 67344 [2], 67432, 69727, 73642, 74232, 74322, 76232, 76323, 76452, 79843 [21], 82263, 83636 [18], 83672 [2], 84755 [2], 85324, 85342, 96236 [18], 97645 [44], 98389 [24] (120), 117722 [2], 123753 [21], 125243 [20], ...
Only four in a row and in brackets the difference if other than one.
3474, 22332, 23324, 23943 [2], 37262, 39482, 52723, 53222 [2], 62394, 63722, 64332, 64792, 67322, 86425 [20] (14), ...
Only five in a row: 2332, 3234, 22372, 22535, 23435, 37482 (6), ...

Only seven in a row: 63432 (1), ...
Note that there is no six Powerful numbers in a row!
The fourth sequence (not an infinite one, I'm sorry) is represented by exponents the same as some combination of digits of the integer: 1364, 3435, 4155, 4316, 4355, 17463, 46712, 48625, (< 60,000), ... , 438579088, ...
The fifth sequence is represented by exponents (in order are under-lined) which are contiguous (rearrengement allowed): 24, 43, 63, 89, 135, 175, 267, 332, 357, 518, 598, 849, 1034, 1073, 1074, 1234, 1306, 1326, 1364, 1672, 1676, 2427, 2537, 2574, 2577, 3295, 3437, 3454, 3457, 4403, 5366, 6424, 6443, 6714, 6793, 6972, 7944, 7964, 10693, 10694, 14369, 14697, 14786, 16363, 17026, 17157, 17407, 18436, 18617, 18697, 18946, 21417, 30340, 30341, 32524, 32816, 32873, 33068, 34498, 34836, 35080, 35081, 36980, 36981, 39687, 43498, 47016, 47690, 47691, 48464, 56982, 59193, 59802, 59339, 59443, 59487, 59683, 59802, ...
Numbers whose exponents are just (different) even numbers: 1255, 1306, 1634, 2355, 3706, 4114, 4194, 4228, 4288, 4339, 4449, 4469, 4644, 6649, 6669, 6934, 7906, 8208, 8224, 8289, 9474, 10694, 13133, 13173, 13933, 19733, 20355, 21235, 21835, 22195, 22315, 22335, 22338, 22395, 22465, 22539, 22864, 22935, 22954, 22978, 23215, 23275, 23232, 23235, 23275, 23295, 23562, 23587, 23835, 23875, 24229, 24289, 24628, 24848, 24868, 25323, 29259, 29835, 32235, 32859, 32922, 38322, 46770, 46771, 48625, 52362, 53262, 58362, 58563, 58962, 59223, 59283, 59823, 59883, ...
Numbers whose exponents are different (in order) odd numbers: 132, 224, 375, 463, 518, 1542, 2048, 2062, 2083, 2114, 2203, 2205, 2240-9, 2315, 2423, 2427, 2733, 2738, 2803, 2953, 3292, 3940-9, 5182, 5320-9, 5343, 5384, 5832, 5902, 8200, 8201, 8316, 8712, 10693, 10723, 12432, 13532, 17026, 17157, 17240-9, 17287, 17352, 17424, 17443, 20843, 21348, 21439, 21564, 22378, 22593, 22953, 24176, 24622, 24680, 24681, 24953, 25127, 25214, 25257, 25327, 26426, 27052, 28423, 32235, 32326, 33022, 33042, 35080-9, 35203, 35932, 36324, 36343, 37226, 38124, 38762, 41727, 42623, 42662, 43212, 49890, 49891, 49922, 52294, 52304, 52344, 59822, 59912, ...
This sequence (not an infinite one, I'm sorry) are those numbers whose exponents are all the same: 153, 370, 371, 407, 1634, 4150, 4151, 8208, 9474, 54748, 92727, 93084, ... ? ..., 194979, ...
Near misses, that is these numbers have exponents that are all the same expect for one and thereby this one possible might be infinite: (24, 43, 63, 89) 132, 135, 153, 175, 209, 226, 262, 264, 283, 334, 370-9, 407, 445, 518, 629, 739, 794, 935, 1034, 1073, 1074, 1255, 1306, 1349, 1386, 1498, 1634, 1765, 1836, 2064, 2114, 2130, 2131, 2139, 2179, 2203, 2245, 2355, 2407,
2517, 2536, 2607, 2932, 3525, 3543, 3612, 3706, 3786, 4114, 4150-9, 4194, 4245, 4405, 4572, 4735, 4950, 4951, 6579, 6649, 6669, 6934, 7906, 8200-9, 8224, 8289, 8316, 8756, 8825, 8864, 8970, 8971, 9272, 9947, 13132, 13133, 13173, 16430, 16431, 16470, 16471, 17133, 17248, 17448, 17840, 17841, 17914, 20203, 20377, 22315, 22243, 23587, 23812, 24848, 25764, 27549, 32780-9, 32810, 32811, 32907, 33442, 34038, 36385, 37002, 37732, 38456, 40608, 41987, 42624, 46770, 46771, 51862, 52700, 52701, 52748, 52768, 52857, 55827, 59060-9, 59353, 59462, 59516, 59606, 59930, 59931, ...
Like any large set of numbers (I'm thinking of the primes here) there are a large, all be it finite, number of subsets. Such is the case here. Unlike the primes, the "powerfuls" are interesting and curious, but are not the fundemental building blocks that the primes represent. Therefore, I shall stop with this next one.
Finally, this sixth sequence (exponents equal to the number of digits/ places of the integer), also know as "Armstrong" or "Pluperfect" numbers, (again, not an infinite one, I'm sorry) is as follows: 153, 370, 371, 407, 1634,

8208, 9474, 54748, 92727, 93084, 548834, 1741725, 4210818, 9800817, 9926315,
24678050, 24678051, 88593477, 146511208, 472335975, 534494836, 912985153,
4679307774, 32164049650, 32164049651, 40028394225, 42678290603, 44708635679,
49388550606, 82693916578, 94204591914, 28116440335967, 4338281769391370,
4338281769391371, 21897142587612075, 35641594208964132, 35875699062250035,
1517841543307505039, 3289582984443187032, 4498128791164624869, 4929273885928088826,
63105425988599693916, 128468643043731391252, 449177399146038697307, 21887696841122916288
27879694893054074471405, 27907865009977052567814, 28361281321319229463398,
35452590104031691935943, and no others less that 2^80, ...

As always, if I extend these series to an appreciable amount, I well forward the same to you.

Reference: Journal of Recreational Mathematics, v10n3p209 by Steven Kahan, Flushing, NY, 1977.

Journal of Recreational Mathematics, v14n1p4-10 by David E. Kullman, Oxford, Ohio, 1981.

"Finding Pluperfect Digital Invariants: Techniques, Results, and Observations," Lionel E. Deimel, Jr., and Michael T. Jones, Journal of Recreational Mathematics, v14n2p87-107, 1982.

Dictionary of Curious and Interesting Numbers, David Wells, Penguin, 1986.

Jean-Pierre Lamoitier, Fifty BASIC Exercises, SYBEX, 1981.

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

13 November 1992

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

In the cited referenced text on page 82, under the recitation for the number eleven is the follow: "Given any 4 consecutive integers greater than 11, there is at least one of them that is divisible by a prime greater than 11." Question! For the other prime numbers, what is the number above which this still holds true? Please find the primes is order on the following page.

Reference: The Penguin Dictonary of Curious and Interesting Numbers, David Wells, Middlesex, England, 1986.

Sequentially yours,

Robert G. Wilson v,
Ph.D., ATP/CF&GI
RGWv:hp110+

2	0
3	1
5	3
7	7
11	9
13	63
17	63
19	168
23	322
29	322
31	1518
37	1518
41	1680
43	10,878
47	17,575
53	17,575
59	17,575
61	17,575
67	17,575
71	17,575
73	70,224
79	70,224
83	97,524
89	97,524
97	97,524
101	97,524
103	224,846
107	224,846
109	612,360
113	688,973
127	688,973
131	688,973
137	688,973
139	688,973
149	688,973

151
157
163
167
173
179
181
191
193
197
199
211
223
227
229

12 October 1993
Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc
600 Mountain Avenue.
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,

In the cited referenced text SI2 on page 27, there is the following theorem numbered three; "If n is a natural number 2, then between n and n there is at least one prime number." This beings up several questions. Only three will be addressed here. One, how many primes are there between n and n ? Or better yet, just how many primes are less than or equal to n factorial? Two, what is the last prime before n factorial? And three, what is the difference between n and the Prime just preceeding it?

n	Last Prime before n	the Diff.	Nbr of Prime:
1	0	0	0
2	0	0	0
3	5	1	3
4	23	1	9
5	113	7	30
6	719	1	128
7	5,039	1	675
8	40,289	31	4,231
9	362,867	13	30,969
10	3,628,789	11	258,689
11	39,916,787	13	2,428,956
12	479,001,599	1	25,306,287
13	6,227,020,777	23	289,620,751
14	87,178,291,199	1	3,610,490,805

n	Last Prime before n!	the D:
15	1,307,674,367,953	47
16	20,922,789,887,947	53
17	355,687,428,095,941	59
18	6,402,373,705,727,959	41
19	121,645,100,408,831,899	101
20	2,432,902,008,176,639,969	31
21	51,090,942,171,709,439,969	31
22	1,124,000,727,777,607,679,927	73
23	25,852,016,738,884,976,639,911	89
24	620,448,401,733,239,439,359,927	73
25	15,511,210,043,330,985,983,999,851	149

From the above table, it is easily discernable that the prime quoted in column two is the nth prime as noted is column four. Therefore; for n 0 the following sequence for (n!) [KN1] is : 0, 0, 1, 3, 9, 30, 128, 675, 4231, 30969, 258689, 2428956, 25306287, 289620751, 3610490805, ~48686917622, ~706003775292, ~10953618190634, ~181035031636349, ~3175094502815204, ~5889360170794762O . The tilde "~" means about and is not necessarily exact! and the hyphen "-" means the number is seperated at that point and continues to the next line, but it is still one number.

Two of the above sequences grow so quickly that they will exceed the two lines given in your forthcoming tome. However, the sequence of the differences between n! and its immediate preceeding prime is restated for index n and from the beginning it is as follows: 0, 0, 1, 1, 7, 1, 1, 31, 13, 11, 13, 1, 23, 1, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 1, 61, 1, 1, 193, 113, 127, 97, 1, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 1, 109, 163, 311, 173, 571, 271, 479, .

In the preceeding series, there are instances where n! 1 is a prime. The following is that sequence for the index n: 3, 4, 6, 7, 12, 14, 30, 32, 33, 38, 94, 166, 324, 379, 469, 546, 974, and no others less than 1156.

Not only is the result of the first difference produce a prime, but so is the result of the difference between the n! and the second preceeding prime, beginning with n =3. They are as follows: 3, 5, 11, 11, 17, 37, 17, 17, 17, 13, 61, 17, 59, 71, 61, 43, 113, 71, 41, 101, 191, 103, 191, 179, 71, 127, 37, 79, 113, 163, 47, 373, 293, 157, 149, 79, 167, 211, 151, 89, 131, 113, 73, 107, 179, 227, 173, 113, 257, 239, 151, 227, 163, 509, 293, 347, 643, 373, 457, 109, 199, 661, 317, 659, 277, 547, 197, 139, 887, 211, 953, 499, 223, 353, 569, 347, 739, 107, 337, 409, 1051, 557, 787, 443, 953, 331, 281, 103, 1031, 467, 409, 449, 127, 613, 467, 607, 619, 839, 631, .

So is the result of the difference between the n! and the third preceeding prime also produces a prime, beginning with n =4. They are as follows: 7, 13, 19, 19, 43, 29, 23, 43, 17, 67, 43, 71, 89, 239, 47, 197, 151, 43, 139, 197, 191, 239, 191, 173, 197, 47, 97, 223, 373, 71, 439, 307, 263, 157, 241, 199, 233, 337, 131, 179, 149, 113, 227, 269, 409, 197, 193, 379, 271, 181, 419, 367, 701, 751, 379, 811, 401, 463, 263, 839, 773, 683, 811, 983, 571, 359, 293, 1039, 281, 1399, 523, 271, 523, 877, 461, 743, 109, 491, 433, 1229, 991, 1987, 661, 1307, 887, 683, 139, 1213, 487, 557, 479, 179, 719, 829, 1103, 1901, 1427, 659, .

What follows is a table of n! until the difference of the previous prime becomes composite.

1! :
2! :
3! : 1, 3, 4=22
4! : 1, 5, 7, 11, 13, 17, 19, 21=37
5! : 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 49=77
6! : 1, 11, 19, 29, 37, 43, 47, 59, 61, 67, 73, 77=77
7! : 1, 17, 19, 29, 31, 37, 41, 47, 53, 67, 71, 73, 83, 89, 97, 103, 107, 109, 121=1111
8! : 31, 37, 43, 67, 79, 83, 89, 107, 127, 131, 143=1113
9! : 13, 17, 29, 79, 121=1111
10! : 11, 17, 23, 37, 41, 53, 89, 103, 121=1111
11! : 13, 17, 43, 73, 83, 89, 97, 113, 149, 151, 163, 167, 199, 247=1319
12! : 1, 13, 17, 37, 61, 73, 83, 107, 139, 169=1313
13! : 23, 61, 67, 89, 101, 103, 109, 163, 191, 193, 223, 227, 233, 269, 281, 283, 289=1717
14! : 1, 17, 43, 47, 61, 67, 89, 131, 197, 199, 211, 227, 233, 313, 347, 353, 359, 373, 419, 449, 461, 499, 601, 607, 659, 667=2329
15! : 47, 59, 71, 79, 89, 97, 139, 157, 163, 167, 227, 251, 257, 293, 313, 317, 347, 367, 401, 419, 449, 461, 463, 467, 493=1729
16! : 53, 71, 89, 107, 139, 163, 181, 227, 239, 263, 307, 317, 337, 353, 359, 367, 421, 431, 433, 449, 457, 467, 563, 569, 571, 587, 601, 641, 673, 677, 701, 739, 743, 757, 787, 911, 947, 967, 983, 1007=1953
17! : 59, 61, 239, 281, 307, 367, 373, 389, 401, 487, 491, 547, 587, 619, 631, 653, 673, 719, 743, 751, 811, 821, 823, 827, 839, 859, 883, 941, 947, 1009, 1151, 1163, 1201, 1271=3141
18! : 41, 43, 47, 61, 101, 109, 131, 139, 157, 193, 257, 283, 311,

   317, 331, 349, 421, 431, 439, 457, 499, 547, 557, 557, 589=1931
19! : 101, 113, 197, 199, 251, 263, 307, 331, 353, 359, 409, 541,
   659, 887, 929, 961=3131
20! : 31, 71, 151, 163, 179, 191, 251, 257, 269, 353, 367, 397, 433,
   443, 521, 601, 613, 631, 641, 659, 661, 797, 823, 853, 857, 859,
   863, 877, 1019, 1063, 1097, 1109, 1171, 1181, 1213, 1229, 1279,
   1291, 1297, 1361, 1423, 1427, 1493, 1537=2953

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 0, 0, 3, 8, 11, 12, 19, 11, 5, 9, 14, 10, 17, 26, 25, 41, 34, 25, 16, 44, 28, 33, 25, 22, 13, 18, 29, 23, 22, 22, 21, 36, 24, 24, 22, 26, 19, 21, 35, 19, 32, 30, 38, 32, 27, 23, 30, 37, 55, 23, 35, 33, 34, 55, 39, 46, 24, 39, 24, 19, 39, 26, 35, 28, 37, 27, 20, 44, 27, 44, 46, 44, 25, 32, 50, 87, 39, 46, 46, 31, 23, 32, 25, 41, 41, 42, 35, 46, 29, 49, 49, 30, 67, 42, 44, 30, 38, 40, 22, 48, 40, .

Just like the preceeding sequences are on the side less that the factorial of n, so do we have sequences on the greater side of n! as well.

The mirror image of the first such sequence is the difference between n factorial and the first prime that exceeds it. Keep in mind that 0! = 1 by definition [Graham p.111] so this sequence begins with the index n = 0.   It is as follows: 1, 1, 1, 5, 7, 7, 11, 23, 17, 11, 1, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 1, 67, 223, 107, 127, 79, 37, 97, 61, 131, 1, 43, 97, 53, 1, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 1, 179, 229, 113, 1, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, .

In the preceeding series, there are instances where n! +1 is a prime. The following is that sequence for the index n: 1, 2, 3, 11, 27, 37, 41, 73, 77, 116, 154, 320, 340, 399, 427, 872, 1477, and no others less than 2043.

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One, just as " I. Chowla [I believe that his initial is S.] has advanced the conjecture that if the number 1 is considered as a prime number (as some people did formerly), " [SI2]   the above sequence (the amount by which the first prime number after n! exceeds n!) contains just primes! These series are somewhat analogous to the "Fortunate" numbers, but I shall leave that sequence to a separate letter.

Here is the one which is analogous to the "Fortunate" numbers using the factorials versus the "primorials:" 2, 3, 5, 5, 7, 7, 11, 23, 17, 11, 17, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 47, 67, 223, 107, 127, 79, 37, 97, 61, 131, 311, 43, 97, 53, 61, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 79, 179, 229, 113, 149, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, .

The preceeding sequence represents the difference between n! 1 and the first prime exceeding n! 1. This one is its mirror image; ie, the difference between n! 1 and the first prime preceeding n! 1 beginning with n =5: 7, 11, 17, 31, 13, 11, 13, 13, 23, 17, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 79, 61, 163, 47, 193, 113, 127, 97, 79, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 449, 109, 163, 311, 173, 571, 271, 479, .

The difference between the two sequences, the first one citing the differences between n! and its immediate preceeding prime and the second one citing the differences between n! and its immediate succeeding prime, is the "prime gap" surrounding the number n! The largest observed difference being about the number 71! and it is equal to 1608, which equals the spread of the pair (71! 751, 71! + 857). Since the two sequences above contain only odd numbers, the differences are always even, therefore; the following series is the difference divided by two, beginning with n = 3: 1, 3, 7, 4, 6, 27, 15, 11, 7, 15, 45, 10, 45, 38, 45, 39, 95, 30, 31, 52, 93, 102, 95, 48, 22, 84, 127, 54, 94, 40, 19, 145, 87, 129, 49, 22, 85, 68, 66, 121, 90, 78, 146, 95, 156, 78, 71, 79, 225, 60, 65, 175, 66, 305, 192, 196, 215, 205, 420, 101, 186, 213, 160, 300, 132, 167, 117, 118, 804, 132, 187, 189, 198, 135, 246, 215, 264, 105, 392, 139, 255, 345, 257, 108, 639, 366, 153, 168, 581, 238, 125, 136, 328, 181, 270, 240, 337, 250, 309, .

Not only is the result of the first difference above n! produce a prime, but so is the result of the difference between the n! and the second succeeding prime. They are as follows: 2, 3, 5, 7, 11, 13, 19, 31, 23, 19, 17, 43, 73, 41, 149, 41, 53, 61, 109, 37, 37, 71, 109, 193, 97, 173, 47, 101, 229, 163, 241, 83, 139, 103, 83, 577, 311, 47, 269, 61, 61, 107, 97, 89, 379, 149, 269, 83, 137, 167, 281, 89, 79, 443, 229, 157, 179, 563, 389, 277, 827, 281, 433, 151, 383, 1033, 79, 101, 137, 173, 971, 151, 79, 479, 449, 139, 149, 307, 839, 139, 757, 971, 227, 919, 229, 509, 593, 787, 1187, 1069, 251, 167, 193, 479, 557, 239, 257, 499, 109, 439, 233, .

So is the result of the difference between the n! and the third succeeding prime after n! also produces a prime. They are as follows: 5, 7, 13, 17, 19, 37, 37, 31, 41, 19, 59, 109, 71, 179, 73, 59, 73, 113, 53, 47, 127, 149, 263, 107, 241, 59, 103, 317, 241, 317, 113, 197, 127, 109, 647, 397, 67, 281, 67, 211, 163, 109, 107, 439, 521, 709, 101, 383, 337, 397, 223, 337, 601, 281, 311, 389, 821, 421, 307, 911, 353, 787, 293, 431, 1051, 233, 461, 241, 307, 991, 251, 107, 487, 997, 151, 197, 647, 881, 157, 937, 1697, 487, 971, 311, 541, 1117, 1129, 1249, 1709, 727, 661, 257, 1153, 607, 461, 337, 523, 239, 547, 563, .

What follows is a table of n! until the difference of n! and the subsequent prime becomes composite.

1! : 1, 2, 4=22
2! : 1, 3, 5, 9=33
3! : 1, 5, 7, 11, 13, 17, 23, 25=55
4! : 5, 7, 13, 17, 19, 23, 29, 35=57
5! : 7, 11, 17, 19, 29, 31, 37, 43, 47, 53, 59, 61, 71, 73, 77=711
6! : 7, 13, 19, 23, 31, 37, 41, 49=77
7! : 11, 19, 37, 41, 47, 59, 61, 67, 73, 79, 107, 113, 127, 131, 139, 149, 157, 169=1313
8! : 23, 31, 37, 41, 67, 103, 107, 109, 113, 139, 151, 163, 167, 173, 179, 187=1117
9! : 17, 23, 31, 47, 61, 71, 73, 89, 97, 103, 107, 137, 139, 157, 163, 167, 179, 181, 187=1117
10! : 11, 19, 41, 47, 53, 83, 97, 127, 131, 149, 167, 169=1313
11! : 1, 17, 19, 29, 31, 37, 59, 109, 113, 131, 139, 149, 173, 179, 191, 211, 227, 239, 169=1313
12! : 29, 43, 59, 83, 101, 137, 167, 191, 193, 197, 221=1317
13! : 67, 73, 109, 139, 157, 181, 199, 289=1717
14! : 19, 41, 71, 79, 103, 127, 167, 193, 229, 353, 361=1919
15! : 43, 149, 179, 223, 227, 361=1919
16! : 23, 41, 73, 149, 163, 173, 197, 199, 233, 277, 283, 323=1719
17! : 31, 53, 59, 83, 109, 167, 181, 263, 281, 283, 293, 307, 349, 367, 397, 421, 487, 599, 631, 739, 761, 769, 779=1941
18! : 37, 61, 73, 101, 103, 107, 137, 173, 179, 239, 241, 257, 271, 293, 331, 337, 383, 397, 463, 541, 613, 617, 683, 713=2331

19! : 89, 109, 113, 137, 139, 241, 281, 311, 389, 463, 467, 641,
659, 701, 713=2331
20! : 29, 37, 53, 79, 89, 137, 139, 157, 167, 211, 271, 277, 283,
353, 367, 379, 439, 491, 571, 617, 619, 661, 673, 743, 839,
919, 941, 953, 997, 1091, 1189=2941

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 3, 4, 8, 8, 15, 8, 18, 16, 19, 12, 19, 11, 8, 11, 6, 12, 23, 24, 15, 31, 21, 27, 15, 16, 26, 25, 18, 17, 29, 20, 27, 27, 30, 23, 16, 28, 24, 25, 29, 15, 25, 19, 36, 36, 39, 15, 36, 24, 44, 35, 29, 27, 25, 36, 22, 37, 31, 32, 41, 29, 55, 27, 45, 29, 59, 34, 37, 24, 49, 25, 40, 29, 55, 39, 38, 34, 46, 31, 37, 41, 24, 25, 35, 45, 33, 41, 42, 63, 31, 49, 46, 40, 30, 28, 36, 50, 36, 26, 32, 31, 37, . Please notice that in the above table for n 0 and in the other table for n 4, the first composite difference has as its prime factors, numbers which exceed n.

Given the sequence immediately above and its counterpart cited earlier, the sum of the two less the two end points states the number of prime differences surrounding n factorial. It is as follows: 2, 3, 9, 14, 24, 18, 35, 25, 22, 19, 31, 19, 23, 35, 29, 51, 55, 47, 29, 73, 47, 58, 38, 36, 37, 41, 45, 38, 49, 40, 46, 61, 52, 45, 36, 52, 41, 44, 62, 32, 55, 47, 72, 66, 64, 36, 64, 59, 97, 56, 62, 58, 57, 89, 59, 81, 53, 69, 63, 46, 92, 51, 78, 55, 94, 59, 55, 66, 74, 67, 84, 71, 78, 69, 86, 119, 83, 75, 81, 70, 45, 55, 58, 84, 72, 81, 75, 107, 58, 96, 93, 68, 95, 68, 78, 78, 72, 64, 52, 77, 75, .

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One; for n 2, there exists at least six primes - three below n! and three above n!; for n 9, there exits at least ten primes - five below n! and five above n!; and for n 15, there exits at least twenty primes - ten below n! and ten above n!; which represent the absolute difference of the consecutive primes and n! Additionally, as n grows so does the lower limit. Take for example, n = 10, there are 3 consecutive primes immediately below 10!, they being 3628777, 3628783 and 3628789, and 3 consecutive primes immediately above 10!, they being 3628811, 3628819 and 3628841. The differeces are 23, 17, 11, +11, +19 and +41, and these are all prime. This is rather astonishing since a cursory look at Table 1 on page 390 of Knuth [KN1] demonstrates the rarity of this condition.

This sequence is n! is as follows: 1, 3, 9, 33, 153, 873, 5913, 46233,
409113, 4037913,
43954713, 522956313, 6749977113, 93928268313, 1401602636313, 22324392524313,
378011820620313, 6780385526348313, 128425485935180313, 2561327494111820313,
53652269665821260313, 1177652997443428940313, 27029669736328405580313, 6474780714695678
16158688114800553828940313, 419450149241406189412940313, 11308319599659758350180940313,
316196664211373618851684940313, 9157958657951075573395300940313, 2744108184701421342097
8497249472648064951935266660940313, 271628086406341595119153278820940313,
8954945705218228090637347680100940313, 304187744744822368938255957323620940313,
10637335711130967298604907294846820940313, 3826306625010321847666043554456820209-40313,
14146383753727377231082583937026584420940313, 5371690012203284889910898080-371008756209
20935051082417771847631371547939998232420940313, 83685033433031550619324264114405589250
342893769474941226143633-04694584807557656420940313, 1439295494700374021157505910939096
61854558558074209658512637979453093884758552420940313, 27201261333465229777021384489940
122342346998826717539665299944651784048588130840420940313, 5624964506810915667389970728
264248206017979096310354325882356886646207872272920420940313, 1267816379855405176717264
620960027832821612639424806694551108812720525606160920420940313, 3103505322954619965625
. This sequence for all n 4 have a digital root of 9 and for all n 10 are divisible by 99.

This sequence is when the above series less two is prime for the index n:

2, 3, 4, 5, 12, 13, 19, 65, 90, 123, .
Notice that the lesser significant digits of n! become identical for increasing modulus powers of ten as n increases. Therefore; the sequence of the digits in reverse order (least to greatest significant digit) is as follows: 3, 1, 3, 0, 4, 9, 0, 2, 4, 0, 2, 9, 8, 2, 5, 6, 3, 3, 2, 4, 4, 6, 5, 5, 2, 5, 0, 9, 3, 0, 5, 0, 1, 3, 9, 5, 3, 2, 3, 4, 0, 8, 4, 9, 9, 7, 0, 1, 1, 2, 6, 8, 3, 7, 4, 8, 6, 8, 7, 4, 9, 7, 4, 7, 4, 2, 2, 9, 0, 0, 4, 3, 3, 0, 5, 6, 5, 8, 6, 5, 0, 0, 2, 6, 6, 5, 1, 5, 9, 7, 8, 8, 1, 6, 2, 0, 2, 8, 1, 2, 1, .
This sequence is the first prime factor of n! which exceeds n, or if prime, then the entry
will be a zero: 1, 1, 1, 11, 17, 0, 0, 11, 131, 11, 0, 23, 821, 2789, 107, 0, 163, 19727, 29, 53, 877, 0, 139, 37, 454823, 107, 431363, 191, 37, 0, 1231547, 41, 109, 0, 7523968684626643, 542410073, 379, 127, 956042657, 0, 1652359939, 0, 134593, 467, 2663, 18701, 0, 12890567, 0, 1361, 659, 331, 1301, 2927, 20676637417223710763l553, 199, 79, 67, 1181, 149382661, 18492619, 312269-44590932009, 627898235566991, 733, 1427427733176073, 887, 211, 0, 17569, 139, 151, 9619, 229, 0, 239, 263, 443, 3023, 0, 8293, ???81???, 627078017207, 97, 4057, 131, 2252681, 163, ???88???, 1327, 30178433, 5107, 11013997953139, 5088213983, 519632149363, 149, 297509, 32869, 296340509, 1607, 168463, 4099, 148303, . The two unknown entries (???n???) at indexes 81 and 88 are primes which exceed 3e9.
And this last sequence is when n! is a prime for the index n: 2, 4, 5, 6, 7, 11, 16,
22, 30, 34, 40, 42, 47, 49, 68, 74, 79, 168, 202, 245, and no others less than or equal to 262. Note that this series needs its own recitation, but notice that for all the above primes, the lesser significant digits become identical with increasing nodulus power of ten as n increases, just as the previous series does. Therefore; all that needs to be done is to take the preceeding sequence and divide it by 99.
What follows is not a series but are just observations in the gap between the primes (the size is followed by the factorial and which primes that surround it): 732=88!1, 750=66!+1+2, 784=81!1, 830=90!+1+2, 840=61!1, 844=82!+1+2, 1078=89!+1+2, 1162=91!1, 1200=85!23, 1278=87!1, 1282=99!23, and lastly 1608=71!1.

Reference: W.W. Rouse Ball and H.S.M. Coxeter, Mathematical Recreations and
Essays, 13th edition, Dover Publ., N.Y., p66
JRM v18n2p89.
J.P. Buhler, R.E. Crandall and M.A. Penk, "Primes of the form n!1
and 2*3*5***p 1," Math. of Comp., v38p639-43, 1982.
Thomas P. Dence, Solving Math Problems in BASIC, p71-2.
Richard K. Guy, Am. Math. Mo. v95n8p699-700&708, Oct 88.
Ronald Lewis Graham, Donald Ervin Knuth, Oren Patashnik, Concrete
Mathematics, A Foundation For Computer Science, Addison-Wesley
Publishing Co., Reading, Mass, 1989.
KN1: Donald Ervin Knuth, Seminumerical Algorithms, "The Art of
Computer Programming," Ed. 2, Vol. 2, Page 365, Addison-Wesley
Publ. Co., Reading, Mass, 1980.
Stephen P. Richards, A Number For Your Thoughts, New Providence, N.J.,
p70&200.
Joe Roberts, Lure of the Integers, The Mathematical Association of
America, 1992, p62.
SI2: Waclaw Sierpinski (1882-1969), A Selection Of Problems in the Theory Of
Numbers, pg. 71, A Pergamon Press Book, the MacMillan Co,. NY, 1964.
David Wells, The Penguin Dictionary of Curious and Interesting Numbers,

p161, d1986.
Stephen Wolfram, "Mathematica," MS-DOS 386/7 Version 2.0.2
released 21 June 1991, the function PrimePi[n!].
Soft Warehouse, "Derive," Version 2.02, 30 Nov 1990,
the function Next_Prime(n!).
Jeff Young & Aaron Potler, with Cray Research, Inc., First Occurrence Prime
Gaps, Math. Of Computation, v52n185p221-4, Jan 89.
Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP / CF&GI
RGWv:T4400C
sloane23

12 October 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following sequences for inclusion in your forthcoming second edition of the above. The first sequence is the most important one and it is named after Frank Gray, at that time an engineer for AT&T Bell Laboratories to poovide ar error-correcting technique for electronic communications.
The sequence begins as follows: 0, 1, 3, 2, 6, 7, 5, 4, 12, 13, 15, 14, 10, 11, 9, 8, 24, 25, 27, 26, 30, 31, 29, 28, 20, 21, 23, 22, 18, 19, 17, 16, 48, 49, 51, 50, 54, 55, 53, 52, 60, 61, 63, 62, 58, 59, 57, 56, 40, 41, 43, 42, 46, 47, 45, 44, 36, 37, 39, 38, 34, 35, 33, 32, 96, 97, 99, 98, 102, 103, 101, 100, 108, 109, 111, 110, 106, 107, 105, 104, 120, 121, 123, 122, 126, 127, 125, 124, 116, 117, 119, 118, 114, 115, 113, 112, 80, 81, 83, 82, 86, 87, 85, 84, 92, 93, 95, 94, 90, 91, 89, 88, 72, 73, 75, 74, 78, 79, 77, 76, 68, 69, 71, 70, 66, 67, .
Since this sequence is best visualized in the binary mode, here it is: 0, 1, 11, 10, 110, 111, 101, 100, 1100, 1101, 1111, 1110, 1010, 1011, 1001, 1000, 11000, 11001, 11011, 11010, 11110, 11111, 11101, 11100, 10100, 10101, 10111, 10110, 10010, 10011, 10001, 10000, 110000, 110001, 110011, 110010, 110110, 110111, 110101, 110100, 111100, 111101, 111111, 111110, 111010, 111011, 111001, 111000, 101000, 101001, 101011, 101010, 101110, 101111, 101101, 101100, 100100, 100101, 100111, 100110, 100010, 100011, 100001, .
And finally the Anti- or Inverse-Gray sequence: 0, 1, 3, 2, 7, 6, 4, 5, 15, 14, 12, 13, 8, 9, 11, 10, 31, 30, 28, 29, 24, 25, 27, 26, 16, 17, 19, 18, 23, 22, 20, 21, 63, 62, 60, 61, 56, 57, 59, 58, 48, 49, 51, 50, 55, 54, 52, 53, 32, 33, 35, 34, 39, 38, 36, 37, 47, 46, 44, 45, 40, 41, 43, 42, 127, 126, 124, 125, 120, 121, 123, 122, 112, 113, 115, 114, 119, 118, 116, 117, 96, 97, 99, 98, 103, 102, 100, 101, 111, 110, 108, 109, 104, 105, 107, 106, 64, 65, 67, 66, 71, 70, 68, 69, 79, 78, 76, 77, 72, 73, 75, 74, 95, 94, 92, 93, 88, 89, 91, 90, 80, 81, 83, 82, 87, 86, 84, 85, 255, .
Unlike the Gray code in binary form, its inverse in binary shows no additional meaning, and therefore; should not be included in the sequences. This is demonstrated by the first few entries, and they are as follows: 0, 1, 11, 10, 111, 110, 100, 101, 1111, 1110, 1100, 1101, 1000, 1001, .

Reference: Scientific America, v251n5p27-8, Nov 1984.
Martin Gardner, Knotted Doughnuts and Other Mathematical Entertainments, "The Binary Gray Code," Chpt. 2, pg11-27, W.H. Freeman and Co., New York, 1986.
Donald Ervin Knuth, The Art Of Computer Programming, 2nd, Ed., Vol. 2, "Seminumerical Algorithns," pgs193&640, Addison-Wesley Publ., Reading, Mass, 1981.
and Vol. 4, "Combinatorial Algorithms," Sec. 7.2.1, [Unpublished. Will it be always so?] [much like Charlie on the MTA]
Wm. H. Press, Numerical Recipes Example Books in FORTRAN, the Art of Scientific Computing, 2nd Ed., pgs 300, 881, 886-8ff, Cambridge University Press, Port Chester, NY, 1992.

Sequentially yours,
Robert G. Wilson v

Ph.D., ATP/CF&GI
RGWv:T4400C
sloane24

14 March 1994

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequence for inclusion in your forth-coming second edition of the above. This sequence is an "important" one since it is named after a very prominent mathe-matician; Donald Ervin Knuth (1938-) of Stanford University.

K0 =1, Kn+1 = 1 + min (2K (n/2(, 3K (n/3()for n (0.

The sequence of Knuth numbers is a non-decreasing one and it begins as follows: 1, 3, 3, 4, 7, 7, 7, 9, 9, 10, 13, 13, 13, 15, 15, 19, 19, 19, 19, 21, 21, 22, 27, 27, 27, 27, 27, 28, 31, 31, 31, 39, 39, 39, 39, 39, 39, 39, 39, 40, 43, 43, 43, 45, 45, 46, 55, 55, 55, 55, 55, 55, 55, 55, 55, 57, 57, 58, 63, 63, 63, 63, 63, 64, 67, 67, 67, 79, 79, 79, 79, 79, 79, 79, 79, 79, 79, 79, 79, 81, 81, 82, 85, 85, 85, 87, 87, 91, 91, 91, 91, 93, 93, 94, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 115, 115, 115, 115, 117, 117, 118, 121, 121, 121, 127, 127, 127, 127, 127, 127, 129, 129, 130, 135, 135, 135, 135, 135, 136, 139, 139, 139, 159, ...

Or if you like, the following is the above sequence without repeats: 1, 3, 4, 7, 9, 10, 13, 15, 19, 21, 22, 27, 28, 31, 39, 40, 43, 45, 46, 55, 57, 58, 63, 64, 67, 79, 81, 82, 85, 87, 91, 93, 94, 111, 115, 117, 118, 121, 127, 129, 130, 135, 136, 139, 159, 163, 165, 166, 171, 172, 175, 183, 187, 189, 190, 193, 202, 223, 231, 235, 237, 238, 243, 244, 247, 255, 256, 259, 261, 262, 271, 273, 274, 279, 280, 283, 319, 327, 331, 333, 334, 343, 345, 346, 351, 352, 355, 364, 367, 375, 379, 381, 382, 387, 388, 391, 405, 406, 409, 418, 447, 463, 471, 475, 477, 478, 487, 489, 490, 495, 496, 499, 511, ...

You should notice and it takes little effort to prove that for all ns (0, that Kn(1 (Kn > n. Also there exists ms and ns such that Kn = Kn(1 = = Kn(m1 = Kn+m = n+m+1 < Kn+m+1 for ever increasing ms. Beginning with m = 0, this series for the ns is as follows: 1, 3, 7, 19, 27, 127, 742, 39, 55, 22153, 4479, 79, 31468, 405, 2323, 463, 111, 33169, 16402, 159, 223, 7573, 5247, 811, ??24??, 3711, 607, 30619, 447, 42525, 1822, 639, 1375, ??33??, ??34??, 319, ??36??, 32805, ??38??, 88615, 1215, 6967, ??42??, 1623, 895, ??45??, ??46??, 3247, 11056, 1791, ??50??, 125875, 10623, 22207, 3766, 6495, 49207, 15673, 132921, 3583, ??60??, 3645, 6682, 2623, 2751, 21247, ??66??, 4095, 2431, ??69??, 63615, 12991, ??72??, ??73??, 10570, 65611, 8703, ??77??, ??78??, 7167, 5224, ??81??, 171519, 5839, ??84??, ??85??, ??86??, ??87??, 85503, ??89??, ??90??, 42495, 5467, ??93??, 89181, 17503, ??96??, 22113, 66559, ??99??, ??100??, 85759, ??102??, 25983, 42751, ??105??, ??106??, 88939, ??108??, 7533, ??110??, 122479, 21375, 1270891, ??114??, 31347, 57343, ??117??, 28671, 14335, ??120??, 10449, 20889, 7291, 4863, 68287, ??126??, 24447, 7807, ??129??, 136575, 8191, ??132??, 62463, ??134??, ??135??, 19455, ...

At no time can K2n = 2Kn, but K2n > 2Kn about of the time. But K3n can equal 3Kn, and K3n > 3Kn about of the time. Like the sequence involving increasing Gaps between the Primes, SSN 984, so are these following two sequences. The first is a sequence whose entries are most often represented up to that point. It is as follows: 1,3,7,19, 27, 39, 55, 79, 111, 159, 223, 319, 895, 1791, 2431, 4863, 7807, 8191, 10935, 14583, 16383, 21871, 29167, 32767, 43743, 65535, 87487, 131071, ...

The second sequence represents the number of entries the preceeding entries are cited. It is as follows: 1, 2, 3, 4, 5, 8, 9, 12, 17, 20, 21, 36, 45, 50, 69, 125, 129, 132, 185, 248, 260, 369, 372, 516, 737, 1028, 1101, 2052, ...

Reference: Ronald Lewis Graham, Donald Ervin Knuth and Oren Patashnik, Concrete Mathematics, A Foundation for Computer Science, p78, Addison-Wesley Publ. Co., Reading, Mass. 1989.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane25

8 April 1994

Neil James Alexander Sloane
% Room 2C(376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue, P.O. Box 636
Murry Hill, New Jersey 07974(0636
908+582(3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Here is the first-order Eulerian Triangle Coefficients which is quite similar to Pascal's Triangle.
SSN 1382: 1, 4, 11, , 33554406, 67108837, 134217700, 268435427, 536870882, 10-73741793, 2147483616, 4294967263, 8589934558, 17179869149, 34359738332, 68719476699, . = 2n (n (1.
SSN 2047: 1, 11, 66, , 41932745, 126781020, 382439924, 1151775897, 3464764515, 10414216090, 31284590870, 93941852511, 282010106381, 846416194536, .
= 3n ((n(1)·2n (C().SSN 2255: 1, 26, 302, , 3572085255, 14875399450, 61403313100, 251732291184, 1026509354985, 4168403181210, 16871482830550, 68111623139600, 274419271461131, .
SSN 2310: 1, 57, 1191, , 85383238549, 473353301060, 2575022097600, 13796160-184500, 73008517581444, 382493246941965, 1987497491971605, 10258045633638475, .
SSN 2336: 1, 120, 4293, , 782115518299, 5717291972382, 40457344748072, 278794377854832, 1879708669896492, 12446388300682056, 81180715002105741, .
SSN 2355: 1, 247, 14608, , 3207483178157, 31055652948388, 285997074307300, 2527925001876036, 21598596303099900, 179385804170146680, 1454842842001939656,
.
SSN 2366: 1, 502, 47840, , 6382798925475, 83137223185370, 1006709967915228, 11485644635009424, 124748182104463860, 1300365805079109480, 13093713503185076040,
.
The next rung down the triangle is: 1, 1013, 152637, 10187685, 423281535, 128432628-63, 311387598411, 6382798925475, 114890380658550, 1865385657780650, 27862280567093-358, 388588260723953310, 5119020713873609970, 64276307695970022450, 7748924718115-06342650, 90224673422637433 68906, 1019558923182105431 72751,
.

A reasoned argument has been put forth that a reference such as this should have as a limit for the second term 1025 instead of 103 or even 210.
"The order 1025 needs a word of explanation. As a matter of fact it was the series of order 1024 that was first completed; the use of a power 2 facilitates the preparation, the three-figure limit is comfortably exceeded, and the series is neither so small as to challenge an early extension nor so large as to be unmanageable. But the more I used the series, the more clearly I saw that its usefulness depended on the ease with which any proposed fraction could be located, either as a term of the series, or if its denominator was above the limit, between consecutive terms. Also unanticipated uniformity in the series was found to imply that location is easiest if the series can be fitted well into a round number of pages. The series of order 1024 occupied nearly 399 pages; extension to the order 1025 required exactly one page more." [Neville]
Also you list the largest number in each row of Pascal's Triangle and name it the "Central Binomial Coefficient" SSN 294. Please consider the following sequence for inclusion in your forthcoming second edition of the above.
In keeping with this same analogy therefore should we not also list the "Central Eulerian Triangular Coefficient?" This series is as follows: 1, 4, 11, 66, 302, 2416, 15619, 156190, 1310354, 15724248, 162512286, 2275172004, 27971176092, 4475-38817472, 6382798925475, 114890380658550, 1865385657780650,

25 August 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forthcoming second edition of the above.

The first is the sequence of the reciprocal of the primes which have for the first time a recurring period of length n. This series is also the first factor of Rn (Repunits), for n>3, not previously used as a factor in Rj, j<n & jn. Furthermore, this sequence is the first occurrence of order n. "[T]he order of such an X modulo d, that is the minimal value of n for which Xn 1 (mod d)." [Bruce] In this case we are using d=10 for the decimal system. The sequence is infinite, with no gaps, begins: 3, R2 = 11, 37, 101, 41, 7, 239, 73, 333667, 9091, 21649, 9901, 53, 909091, 31, 17, 2071723, 19, R19 = 1111111111111111111, 3541, 43, 23, R23 = 1111111111111111111
99990001, 21401, 859, 757, 29, 3191, 211, 2791, 353, 67, 103, 71, 999999000001, 2028119, 90909090-9090909091, 900900900900990990990991, 1676321, 83, 127, 173, 89, 238681, 47, 35121409, 9999999900000001, 505885997, 251, 613, 521, 107, 70541929, 1321, 7841, 21319, 59, 2559647034361, 61, 733, 909090909090909090909-0
10837, 19841, 162503518711, 599144041, 493121, 28559389, 277, 4147571, 2415731423936276
3169, 12171337159, 7253, 151, 722817036322379041, 5237, 157, 317, 5070721, 163, 2670502781396266997, 3367147378267, 226549, 262533041, 57009401, 4003, 617, 497867, 29611, 547, 1289, 9009009009009009009009009009009900990990990990990990990990
6299, 191, 97, 12004721, 197, 199, 60101, 4531530181816613234555190841, 52986961, 1031, 158081, 30703738801, 909090909090909090909090909090-909090909090909090
643, 109, 1192679, 331, 3055705151864730 7, 113, 227, 21319, 31511, 349, 240396841140769, 1889, 923441, 10000999999899-989999000000010001, 15973, 81131, 1811791, 2049349, 751, 5274739, 18797,   .

This second sequence is the above numbers less one divided by n and it is as follows: 2, 5, 12, 25, 8, 1, 34, 9, 37074, 909, 1968, 825, 4, 64935, 2, 1, 121866, 1, 58479532163742690, 177, 2, 1, 48309178743961352-6570, 4166250, 856, 33, 28, 1, 110, 7, 90, 11, 2, 3, 2, 27777750000, 54814, 23923444976076555, 23100023100025410025410, 41983, 2, 3, 4, 2, 5304, 1, 747264, 20833331250000, 10324204, 5, 12, 10, 2, 1306332, 24, 140, 374, 1, 4338384804, 1, 12, 14662756598240469
172, 310, 2500054134, 9077940, 7360, 419991, 4, 59251, 340243862526236159967 54568-88, 44, 166730646, 5711487482898, 2, 9510750477926040, 68, 2, 4, 63384, 2, 325671070901983
40568040702, 2697, 3088624, 662900, 46, 7, 5594, 329, 6, 14, 96871064613000968710646130
67, 2, 1, 123760, 2, 2, 601, 448666354635308241045068 40, 519480, 10, 1520, 292416560, 857632933104631217838765008576329331046312178387 65, 6, 1, 10942, 3, 275288752420246, 1, 2, 187, 274, 3, 2054673855904, 16, 7760, 833416666658332499916660
132, 665, 14730, 16527, 6, 41863, 148,   .

This sequence taken from the above has a one at n for n equaling: 6, 16, 18, 22, 28, 46, 58, 60, 96,   , and this sequence has a two for n equaling: 1, 15, 21, 33, 35, 41, 44, 53, 75, 78, 81, 95, 98, 99,   . These will show up later (as the above cited numbers times n plus 1) in the series ten sequence entitled the Primes of n Cyclic.

And this sequence, numbered three, is the first occurrence from the second sequence and it is a bit harder to generate and to explain from the other ones just stated. For the most part, the number presented at the nth position will be the first number in sequence ten for the nth Cyclic. In any case, you will find it in the nth Cyclic series. Often it is the second number which qualifies but this is not always the case, as with n=121 were the third entry is necessary to meet the criteria. Recall that an n Cyclic is defined as n=(p-1)/Order(p), n will always be a whole number of any prime p and will also be a divisor (not necessarily a prime or a proper factor) of p-1. Therefore n*Order(p)+1=p. However, if there exists a k<n for which the order of k*Order(p)+1=p, then p cannot be the first occurrence at n. It is as follows: 6, 1, 34, 13, 2, 91, 30, 5, 8, 52, 32, 3, 184, 92, 766, 118, 554, 137, 852, 156, 512, 226, 482, 55, 4, 275, 2128, 27, 1332, 377, 3666, 449, 26, 7, 788, 267, 6154, 362, 870, 229, 2900, 161, 424, 72, 1972, 87, 1818, 406, 1758, 803, 908, 2374, 2552, 405, 1774, 3083, 1368, 375, 1034, 514, 456, 144, 3472, 1995, 818, 1648, 94, 77, 3118, 169, 10346, 655, 1956, 1229, 3892, 1780, 2496, 347, 340, 906, 3096, 859, 5082, 149, 3052, 453, 964, 1561, 612, 31, 11032, 1631, 10306, 3498, 6134, 1275, 6438, 74, 8572, 2266, 6312, 351, 31596, 732, 3092, 1528, 6564, 356, 28008, 29, 4546, 535, 752, 407, 5332, 1137, 484, 4381, 25464, 813, 24730, 635, . If you compare this sequence modified by the above stated formula, you will arrive at the fifth sequence except as noted. This situation occurs for the following ns: 6, 10, 14, 15, 17, 20, 21, 27, 30, 31, 32, 37, 44, 47, 50, 52, 54, 55, 56, 60, 63, 64, 69, 71, 72, 75, 76, 78, 80, 81, 82, 83, 88, 91, 92, 93, 94, 96, 97, 99, 100, 101, 103, 104, 107, 109, 111, 115, 116, 118, 121, 124, 125, 130, 133, 135, 138, 143, 147, 149, 150, 151, 152, 154, 155, 156, 157, 159, 161, 162, 163, 166, 167, 168, 170, 171, 178, 184, 186, 190, 191, 193, 194, 195, 196, 197, 198, 200, 201, 204, 205, 206, 207, 208, 211, 214, 216, 218, 221, 230, 231, 232, 236, 239, 240, 242, 243, 244, 247, 249, . And for this series, the ks: 4, 1, 11, 2, 9, 11, 1, 2, 2, 18, 9, 1, 11, 3, 2, 21, 8, 1, 2, 1, 3, 3, 15, 15, 7, 21, 12, 4, 30, 3, 2, 3, 72, 3, 2, 2, 2, 2, 10, 1, 1, 5, 10, 18, 5, 9, 27, 49, 2, 3, 1, 2, 5, 4, 7, 6, 8, 1, 145, 80, 30, 2, 54, 4, 2, 6, 3, 16, 2, 35, 18, 12, 11, 78, 2, 15, 24, 6, 1, 4, 38, 1, 2, 9, 6, 2, 2, 30, 9, 4, 1, 6, 1, 6, 187, 4, 16, 26, 1, 6, 30, 34, 40, 61, 2, 2, 3, 1, 1, 1, .

This fourth sequence in fact is taken from the sequence representing the number of primitive co-factors of Rn. And except for n =3, it is also the number of primes having a Period length of n, and it is as follows: 1, 2, 1, 1, 2, 2, 2, 2, 1, 1, 2, 1, 3, 1, 2, 2, 2, 2, 1, 2, 3, 3, 1, 1, 3, 2, 2, 3, 5, 3, 3, 5, 2, 3, 3, 1, 3, 1, 1, 2, 4, 3, 4, 3, 2, 4, 2, 1, 2, 3, 4, 2, 4, 2, 3, 2, 3, 2, 2, 3, 7, 1, 5, 4, 2, 2, 3, 3, 3, 2, 2, 2, 3, 3, 3, 2, 4, 4, 6, 2, 5, 2, 3, 2, 3, 3, 3, 2, 5, 3, 6, 3, 1, 3, 5, 4, ? it is between 3 and 8 for n =97 ?, 2, 4, 4, . And this sequence is where the above sequence above is equal to one and therefore; the index of the Only Primes with Period length of n, it will include the Prime Repunits, and it is as follows: 1, 2, 3, 4, 9, 10, 12, 14, 19, 23, 24, 36, 38, 39, 48, 62, 93, 106, 120, 134, 150, 196, 294, 317, 586, 597, ?, 1031, . Once the question marks occur in the preceding lists, you are not guaranteed that there are not any intervening entries after that point.

The fifth sequence are those primes which have for the first time a period of length n =(p-1)/order(p) and thus of n cyclic. The sequence is infinite, with no gaps, and it begins: 7, 3, 103, 53, 11, 79, 211, 41, 73, 281, 353, 37, 2393, 449, 3061, 1889, 137, 2467, 16189, 641, 3109, 4973, 11087, 1321, 101, 7151, 7669, 757, 38629, 1231, 49663, 12289, 859, 239, 27581, 9613, 18131, 13757, 33931, 9161, 118901, 6763, 18233, 1409, 88741, 4003, 5171, 19489, 86143, 23201, 46309, 98801, 135257, 271, 2531, 4201, 77977, 21751, 61007, 15361, 27817, 8929, 168211, 2689, 53171, 108769, 6299, 5237, 401029, 11831, 115589, 2161,

2161, 142789, 90947, 142501, 4637, 192193, 6163, 26861, 30161, 100927, 6397, 176459, 12517, 259421, 38959, 83869, 57641, 54469, 2791, 228593, 1933, 68821, 65707, 582731, 101281, 90017, 7253, 251263, 129001, 5051, 35803, 2368589, 25169, 324661, 161969, 400823, 38449, 131891, 3191, 429127, 59921, 84977, 46399, 356501, 9397, 56629, 4013, 3030219, 97561, 1065527, 77471, 475273, 18973, 819251, 177787, 408433, 160001, 321469, 10271, 480509, 15973, 28463, 99563, 111781, 544001, 1672771, 51199, 4079651, 7841, 615607, 110477, 69499, 492769, 321611, 618311, 732943, 18797, 312007, 119551, 969421, 61561, 205633, 139987, 139501, 14197, 24179, 172853, 3270313, 134401, 498779, 162649, 261127, 294053, 107251, 71879, 2009011, 70729, 2586377, 418031, 280099, 10837, 1106509, 143551, 2403451, 473089, 3541, 957107, 1613149, 121321, 51767, 591319, 345139, 785129, 126541, 4093, 70313, 117877, 704593, 77711, 1065017, 773569, 905171, 196523, 522211, 17837, 74861, 82963, 1038383, 75401, 639181, 351077, 3793259, 39373, 85691, 224129, 1427887, 56369, 548417, 148471, 79337, 1176601, 1089709, 206083, 845381, 74521, 1919149, 59951, 947833, 133321, 4444753, 265957, 158777, 641761, 31051, 258319, 4741577, 1232797, 4545193, 329591, 725341, 1348849, 8097217, 478999, 1903501, 273997, 49297, 77351, 233743, 44641, 3931229, 972599, 1098847, 236681, 784981, 248707, 459421, 629921, 861541, 676751, .

Or we may sort the above in numerical order to produce the following: 7, 11, 13, 37, 41, 53, 73, 79, 101, 103, 137, 211, 239, 271, 281, 353, 449, 641, 757, 859, 1231, 1321, 1409, 1889, 1933, 2161, 2393, 2467, 2531, 2689, 2791, 3061, 3109, 3191, 3541, 4003, 4013, 4093, 4201, 4637, 4649, 4973, 5051, 5171, 5237, 6163, 6299, 6397, 6763, 7151, 7253, 7669, 7841, 8779, 8929, 9091, 9161, 9397, 9613, 9901, 10271, 10837,
11087, 11831, 12289, 12517, 13757, 14197, 15361, 15973, 16189, 16763, 17837, 18131, 18233, 18797, 18973, 19489, 19841, 21319, 21401, 21649, 21751, 23201, 23311, 24179, 25169, 25601, 26861, 27581, 27817, 27961, 28463, 29611, 30161, 31051, 31511, 33931, 34849, 35803, 38237, 38449, 38629, 38861, 38959, 39373, 42043, 43037, 44641, 45613, 46309, 46399, 49297, 49663, 51199, 51767, 52009, 52579, 53171, 54469, 56369, 56629, 57641, 59281, 59921, 59951, 60101, 61007, 61561, 62003, 62921, 63799, 63841, 65707, 68389, 68821, 69499, 69857, 70313, 70729, 71879, 72559, 74521, 74687, 74861, 75401, 77351, 77471, 77711, 77977, 79337, 80173, 81131, 82963, 83869, 84977, 85691, 86143, 87211, 88741, 90017, 90679, 90947, 97561, 98641, 98801, 99563, 100927, .

The sixth sequence is the same as the unsorted third, but this time, we are not restricted to the primes (the composites are highlighted and underlined). It is as follows: 7, 13, 103, 53, 11, 79, 211, 41, 73, 281, 353, 37, 2393, 449, 91, 33, 137, 2467, 7107, 641, 3109, 4973, 11087, 1321, 101, 7151, 7669, 757, 38629, 1231, 49663, 12289, 859, 239, 561, 9613, 18131, 13757, 703, 9161, 118901, 6763, 259, 1409, 451, 4003, 5171, 19489, 99, 23201, 46309, 98801, 135257, 271,
2531, 4201, 77977, 21751, 61007, 15361, 27817, 8929, 168211, 2689, 53171, 108769, 6299, 5237, 215143, 11831, 115589, 2161, 142789, 90947, 142501, 4637, 192193, 6163, 26861, 481, 110927, 657, 176459, 12517, 259421, 38959, 83869, 57641, 54469, 2791, 228593, 1933, 68821, 2821, 582731, 1729, 90017, 7253, 251263, 129001, 5051, 35803, 2368589, 25169, 324661, 161969, 400823, 38449, 131891, 3191, 429127, 59921, 84977, 46399, 356501, 9397, 56629, 4013, 3030217, 97561, 1065527, 77471, 475273, 18973, 819251, 177787, 408433, 160001, 52633, 5461, 480509, 15841, 28463, 99563, 111781, 544001, 1672771, 24013, 4079651, 7841, 615607, 6533, 69499, 492769, 321611, 321201, 732943, 18797, 312007, 119551, 969421, 61561, 205633, 1233, 139501, 14197, 24179, 172853, 12403, 134401, 498779, 162649, 261127, 294053, 107251, 71879, 2009011, 70729, 2586377, 418031, 280099, 10837, 1106509, 143551, 2403451, 473089, 3541, 957107, 1613149, 121321, 51767, 591319, 345139, 785129, 126541, 4093, 70313, 117877, 704593, 77711, 1065017, 773569, 905171, 196523, 522211, 17837, 74861, 82963, 19503, 75401, 639181, 351077, 3793259, 39373, 85691, 224129, 4141, 56369, 548417, 148471, 79337, 1176601, 1089709, 99297, 845381, 74521, 1919149, 6541, 947833, 133321, 2165801, 265957, 51291, 641761, 31051, 258319, 909,

1232797, 4545193, 69921, 725341, 1348849, 8097217, 478999, 1903501, 273997, 49297, 77351, 233743, 44641, 3930229, 972599, 1098847, 236681, 14701, 248707, 459421, 629921, 7471, 676751,   .
Or we may sort the above to produce the following: 7, 11, 13, 33, 37, 41, 53, 73, 79, 91, 99, 101, 103, 137, 211, 239, 259, 271, 281, 353, 449, 451, 481, 561, 641, 657, 703, 757, 859, 1231, 1233, 1321, 1409, 1729, 1933, 2161, 2393, 2467, 2531, 2689, 2791, 2821, 3109, 3191, 4003, 4013, 4201, 4637, 4973, 5051, 5171, 5237, 5461, 6163, 6299, 6533,
6763, 7107, 7151, 7253, 7669, 7841, 8929, 9161, 9397, 9613, 11087, 11831, 12403, 12517, 12289, 13757, 14197, 15361, 15841, 18131, 18797, 18973, 19489, 21751, 23201, 24013, 24179, 25169, 26861, 27817, 28463, 35803, 38449, 38629, 38959, 46309, 46399, 49663, 52633, 53171, 54469, 56629, 57641, 59921, 61007, 61561, 68821, 69499, 77471, 77977, 83869, 84977, 90017, 90947, 97561, 98801, 99563, 108769, 110927, 111781, 115589, 118901, 119551, 129001, 131891, 134401, 135257, 139501, 142501, 142789, 160001, 161969, 168211, 172853, 176459, 177787, 192193, 205633, 215143, 228593, 251263, 259421, 312007, 321201, 321611, 324661, 356501, 400823, 408433, 429127, 475273, 480509, 492769, 544001, 582731, 615607, 732943, 819251, 969421, 1065527, 1672771,   .
The seventh sequence is the index where sequence five and sequence six are different, ie, sequence five is represented by a composite number at the nth position. It is as follows: 15, 16, 19, 35, 39, 43, 45, 49, 80, 82, 94, 96, 129, 130, 132, 138, 142, 146, 154, 159, 199, 207, 214, 218, 221, 223, 227, 230, 245, 249,   . These are the numbers: 91, 33, 7107, 561, 703, 259, 451, 99, 481, 657, 2821, 1729, 52633, 5461, 15841, 24013, 6533, 321201, 1233, 12403, 19503, 4141, 99297, 6541, 2165801, 51291, 909, 69921, 14701, 7471,   .
This eighth sequence is the sorted first occurrence of n cyclics of just the composite numbers. It is as follows: 9, 33, 91, 99, 259, 451, 481, 561, 657, 703, 909, 1233, 1729, 2821, 4141, 5461, 6533, 6541, 6601, 7107, 7471, 12403, 12801, 13833, 14701, 15841, 19503, 24013, 34113, 34133, 51291, 52633, 69921, 97681, 99297,   .
This ninth sequence is the order of all odd ns / 0 mod 5, ie n / 10. It is as follows: 2, 6, 2, 2, 6, 16, 18, 6, 22, 3, 28, 15, 2, 3, 6, 5, 21, 46, 42, 16, 13, 18, 58, 60, 6, 33, 22, 35, 8, 6, 13, 9, 41, 28, 44, 6, 15, 96, 2, 4, 34, 53, 108, 3, 112, 6, 48, 22, 5, 42, 21, 130, 18, 8, 46, 46, 6, 42, 148, 75, 16, 78, 13, 66, 81, 166, 78, 18, 43, 58, 178, 180, 60, 16, 6, 95, 192, 98, 99, 33, 84, 22, 18, 30, 35,   .
The tenth sequence is really several series:
Cyclic number of the Primes (not included are the Primes 2 & 5): 2, 1, 5, 2, 1, 1, 1, 1, 2, 12, 8, 2, 1, 4, 1, 1, 2, 2, 9, 6, 2, 2, 1, 25, 3, 2, 1, 1, 3, 1, 17, 3, 1, 2, 2, 2, 1, 4, 1, 1, 2, 1, 2, 2, 7, 1, 2, 1, 1, 34, 8, 5, 1, 1, 1, 54, 4, 10, 2, 2, 2, 2, 1, 4, 3, 1, 2, 3, 11, 2, 1, 2, 1, 1, 1, 4, 2, 2, 1, 3, 2, 1, 2, 2, 14, 3, 1, 3, 2, 2, 1, 1, 1, 1, 1, 10, 2, 1, 6, 2, 2, 2, 1, 1, 2, 1, 2, 2, 3, 12, 7, 1, 2, 20, 6, 1, 2, 1, 3, 3, 2, 2, 3, 1, 1, 2, 1, 12, 3, 1, 6, 28, 2, 4, 4, 2, 4,   . The first occurrence of a particular number in this series is cited in the fifth sequence.
Cycle One: 7, 17, 19, 23, 29, 47, 59, 61, 97, 109, 113, 131, 149, 167, 179, 181, 193, 223, 229, 233, 257, 263, 269, 313, 337, 367, 379, 383, 389, 419, 433, 461, 487, 491, 499, 503, 509, 541, 571, 577, 593, 619, 647, 659, 701, 709, 727, 743, 811, 821, 823, 857, 863, 887, 937, 941, 953, 971, 977, 983, 1019, 1021, 1033, 1051, 1063, 1069,   . This series is also represented by primes with +10 as a primitive root and is presented as SSN 1823.
E. Artin's const. A = 0.37395 58136 19202 28805 47280 54346 41641 51116 29249   . See Mr. Yate's second reference for the applicable use of this constant to the above sequence. In a nutshell, it says that about three eighths of all primes have full length reciprocals; ie, they are "Cycle

One" primes.
Primes with -10 as a primitive root: 3, 17, 29, 31, 43, 61, 67, 71, 83, 97, 107, 109, 113, 149, 151, 163, 181, 191, 193, 199, 227, 229, 233, 257, 269, 283, 307, 311, 313, 337, 347, 359, 389, 431, 433, 439, 443, 461, 467, 479, 509, 523, 541, 563, 577, 587, 593, 599, 631, 683, 701, 709, 719, 787, 821, 827, 839, 857, 883, 911, 919, 937, 941, 947, 953, 977, 991, 1021, .

Primes with both +10 as a primitive root: 17, 29, 61, 97, 109, 113, 149, 181, 193, 229, 233, 257, 269, 313, 337, 389, 433, 461, 509, 541, 577, 593, 701, 709, 821, 857, 937, 941, 953, 977, 1021, 1033, 1069, 1097, 1109, 1153, 1181, 1193, 1217, 1229, 1297, 1301, 1381, 1429, 1433, 1549, 1553, 1621, 1697, 1709, 1741, 1777, 1789, 1861, 1873, 1913, 1949, 2017, 2029, .

Cycle Two: 3, 13, 31, 43, 67, 71, 83, 89, 107, 151, 157, 163, 191, 197, 199, 227, 283, 293, 307, 311, 347, 359, 373, 401, 409, 431, 439, 443, 467, 479, 523, 557, 563, 569, 587, 599, 601, 631, 653, 677, 683, 719, 761, 787, 827, 839, 877, 881, 883, 911, 919, 929, 947, 991, 1039, 1049, 1117, 1123, 1129, 1151, 1163, 1187, 1277, 1279, 1283, 1307, 1319, 1361, 1373, 1399, .

Cycle Three: 103, 127, 139, 331, 349, 421, 457, 463, 607, 661, 673, 691, 739, 829, 967, 1657, 1669, 1699, 1753, 1993, 2011, 2131, 2287, 2647, 2659, 2749, 2953, 3217, 3229, 3583, 3691, 3697, 3739, 3793, 3823, 3931, 4273, 4297, 4513, 4549, 4657, 4903, 4909, 4993, 5011, 5023, 5101, 5113, 5407, 5647, 5851, 5953, 6091, 6229, 6379, 6421, 6451, 6577, 6607, .

Cycle Four: 53, 173, 277, 317, 397, 769, 773, 797, 809, 853, 1009, 1013, 1093, 1493, 1613, 1637, 1693, 1721, 2129, 2213, 2333, 2477, 2521, 2557, 2729, 2797, 2837, 3329, 3373, 3517, 3637, 3733, 3797, 3853, 3877, 4133, 4241, 4253, 4373, 4493, 4729, 4733, 4877, 5081, 5333, 5437, 5477, 5569, 5693, 5717, 5801, 5849, 6133, 6277, 6361, 6449, 6569, 6689, .

Cycle Five: 11, 251, 1061, 1451, 1901, 1931, 2381, 3181, 3491, 3851, 4621, 4861, 5261, 6101, 6491, 6581, 6781, 7331, 8101, 9941, 10331, 10771, 11251, 11261, 11411, 12301, 14051, 14221, 14411, 15091, 15131, 16061, 16141, 16301, 16651, 16811, 16901, 17021, 18371, 18541, 18701, 18731, 19211, 19301, 20341, 20731, 20771, 20981, 21061, 21221, 21341, 21491, 22091, 22621, .

Cycle Six: 79, 547, 643, 751, 907, 997, 1201, 1213, 1237, 1249, 1483, 1489, 1627, 1723, 1747, 1831, 1879, 1987, 2053, 2551, 2683, 3049, 3253, 3319, 3613, 3919, 4159, 4507, 4519, 4801, 4813, 4831, 4969, 5119, 5443, 5557, 5791, 6079, 6151, 6271, 6373, 6427, 6529, 6547, 6907, 7027, 7351, 7603, 7723, 8089, 8191, 8599, 8803, 8923, 9133, 9151, 9283, 9403, .

Cycle Seven: 211, 617, 1499, 2087, 2857, 6007, 6469, 7127, 7211, 7589, 9661, 10193, 13259, 13553, 14771, 18047, 18257, 19937, 20903, 21379, 23549, 26153, 27259, 27539, 32299, 33181, 33461, 34847, 35491, 35897, 41651, 42407, 42491, 43051, 43793, 44269, 44633, 45767, 46229, 47699, 47741, 49057, 49939, 50891, 51647, 55021, 55819, 55903, 56701, 59263, 59753, 60859, 63127, .

Cycle Eight: 41, 241, 1601, 1609, 2441, 2969, 3041, 3449, 3929, 4001, 4409, 5009, 6089, 6521, 6841, 8161, 8329, 8609, 9001, 9041, 9929, 13001, 13241, 14081, 14929, 16001, 16481, 17489, 17881, 18121, 19001, 20249, 20641, 20921, 21529, 22481, 23801, 24169, 24809, 24889, 26041, 26729, 26801, 26881, 26921, 27241, 27529, 28001, 30089, 30809, 32969, 33049, 33641, 34961, .

Cycle Nine: 73, 1423, 1459, 2377, 2503, 3457, 7741, 9433, 10891, 10909, 16057, 17299, 17623, 20269, 21313, 22699, 24103, 26263, 28621, 28927, 29629, 30817, 32257, 34273, 34327, 35461, 35731, 36343, 36793, 37549, 37567, 37657, 38737, 39367, 39979, 40429, 43633, 48673, 49069, 49393, 50023, 50221, 51949, 58771, 59221, 59743, 62659, 63901, 64189, 64621, 64927, 65701, 67699, .

Cycle Ten: 281, 521, 1031, 1951, 2281, 2311, 2591, 3671, 5471, 5711, 6791, 7481, 8111, 8681, 8761, 9281, 9551, 10601, 11321, 12401, 13151, 13591, 14831, 14951, 15671, 16111, 16361, 18671, 21191, 21521, 21881, 24281, 24551, 25391, 25801, 25841, 26161, 26431, 26591, 26711, 28031, 28151, 28591, 29231, 29881, 30881, 33071, 33151, 35201, 36761, 36871, 38231, 42391, 43391, .

Cycle Eleven: 353, 3499, 10429, 13619, 15269, 20091, 25741, 30713, 35509, 38567, 45233, 49171, 57179, 57223, 60149, 63691, 63977, 67783, 77023, 85229, 88463, 90619, 91367, 93941, 96779, 108967, 109913, 110221, 112069, 115259, 117503, 120473, 120847, 121727, 126743, 132331, 135851, 137633, 138469, 143419, 144167, 150833, 151537, 152219, 159457, 159589, 163417, 167971, 175781, 176903, 185021, 190367, 193447, 199783, 200927, 202291, 203017, 204623, 208099, 209771, 218417, 220771, 224863, 226007, 231519, .

Cycle Twelve: 37, 613, 733, 1597, 2677, 3037, 4957, 5197, 5641, 7129, 7333, 7573, 8521, 8521, 8677, 11317, 14281, 14293, 15289, 15373, 16249, 17053, 17293, 17317, 19441, 20161, 21397, 21613, 21997, 23053, 23197, 24133, 25357, 25717, 26053, 26293, 27277, 27397, 29437, 29569, 30649, 31081, 31237, 31477, 33721, 35437, 35533, 37561, 37813, 38557, 40609, 40933, 42013, .

Primes with odd cycles: 7, 11, 17, 19, 23, 29, 47, 59, 61, 73, 97, 101, 103, 109, 113, 127, 131, 137, 139, 149, 167, 179, 181, 193, 211, 223, 229, 233, 251, 257, 263, 269, 313, 331, 337, 349, 353, 367, 379, 383, 389, 419, 421, 433, 457, 461, 463, 487, 491, 499, 503, 509, 541, 571, 577, 593, 607, 617, 619, 647, 659, 661, 673, 691, 701, 709, 727, 739, 743, 811, 821, .

Primes with even cycles: 3, 13, 31, 37, 41, 43, 53, 67, 71, 79, 83, 89, 107, 151, 157, 163, 173, 191, 197, 199, 227, 239, 241, 271, 277, 281, 283, 293, 307, 311, 317, 347, 359, 373, 397, 401, 409, 431, 439, 443, 449, 467, 479, 521, 523, 547, 557, 563, 569, 587, 599, 601, 613, 631, 641, 643, 653, 677, 683, 719, 733, 751, 757, 761, 769, 773, 787, 797, 809, 827, 839, .

These two sequences run fairly even in the number of primes in each group. This "race" is analogous to that of the primes of the two forms 4*k+1. This sequence, numbered eleven, then is when the "lead changes hands" or when plotted on a Cartesian plane the Y=X line is crossed. It is as follows: 3, 11, 83, 103, 919, 967, 1523, 1543, 5641, 5651, 5717, 5741, 9293, 9371, 9403, 9497, 9521, 10501, 10631, 10663, 10733, 10753, 11489, 11497, (and no others less than 100,000), . For all the Primes below one hundred thousand, the lead belongs to the odds over the evens by a 4812 versus 4778 for a margin of just 34. It seems that the "odds" have it, at least at the lower level. The sequence continues: 104947, 105983, 106013, 106087, 106163, 106207, 106397, 106417, 107609, 107713, 107719, 108247, 108677, 108739, 108761, 112337, 112403 112459,
114067, 114269, 114281, 114407, 114689, 114713, 114773, 114967, 114997, 115061, 115079, 115337, 115399, 115429, 115631, 115807, 115853, 115861, 115883, 115901, 116041, 116101, 116933, 116993, 117053, 118709, 118751, 118927, 119359, 119419, 119591, 119617, 119809, 120817, 122449, 122819, 122849, 122887, 123083, 123113, 123203, 123269, 123307, 123593, 123637, 123983, 124133, 124171, 140363, 140411, 140557, 140663, 140681, 147709, 152147, 152189, 152203, 152419, 152729, 152767, 154043, 154061, 154081, 154097, 154373, 154571, 154787, 154823, 154849, 154981, 155003, 155171, 155201, 155299, 155809, 155833, 172399, 172421, 172489, 172709, 172721, 173531, .

This twelfth sequence of Primes is when the number of even Cyclics equals the odd Cyclics and this occurs at: 2, 7, 13, 43, 53, 71, 79, 101, 107, 809, 911, 941, 953, 1013, 1049, 1493, 1511, 1531, 5573, 5591, 5639, 5647, 5653, 5693, 5711, 5737, 5849, 9283, 9349, 9397, 9421, 9433, 9463, 9473, 9491, 9511, 10343, 10499, 10627, 10657, 10667, 10729, 10739, 10889, 11483, 11491, 22159,
104933, 104953, 104971, 105023, 105167, 105269, 105341, 105389, 105509,

105527, 105977, 105997, 106019, 106033, 106129, 106181, 106187, 106213, 106391, 106411, 106427, 106441, 106453, 107603, 107621, 107647, 107687, 107699, 107717, 107741, 108233, 108293, 108359, 108413, 108439, 108649, 108727, 108751, 110273, 110291, 112103, 112331, 112397, 112429, 112481, 112507, 112559, 112573, 114043, 114073, 114259, 114277, 114299, 114377, 114649, 114679, 114691, 114769, 114781, 114941, 114973, 115057, 115067, 115099, 115309, 115331, 115363, 115421, 115613, 115663, 115793, 115849, 115859, 115879, 115891, 116027, 116047, 116099, 116107, 116359, 116923, 116929, 116953, 116989, 117043, 117109, 117127, 117731, 118691, 118717, 118747, 118903, 118913, 119267, 119293, 119321, 119417, 119569, 119611, 119627, 119653, 119677, 119689, 119759, 119773, 119797, 120383, 120811, 121169, 122443, 122527, 122789, 122827, 122839, 122861, 122869, 122929, 122963, 123001, 123049, 123077, 123091, 123121, 123191, 123229, 123259, 123289, 123583, 123601, 123631, 123953, 123979, 124001, 124067, 124123, 124139, 124153, 124277, 140333, 140351, 140381, 140407, 140521, 140533, 140551, 140617, 140629, 140659, 140677, 147073, 147139, 147661, 147673, 147703, 147853, 147919, 152123, 152183, 152197, 152417, 152723, 152753, 154027, 154057, 154067, 154079, 154087, 154111, 154159, 154279, 154369, 154487, 154501, 154543, 154573, 154613, 154667, 154681, 154747, 154769, 154789, 154807, 154841, 154943, 154991, 155069, 155153, 155167, 155191, 155291, 155723, 155773, 155801, 155821, 155849, 172093, 172169, 172213, 172373, 172411, 172441, 172673, 172687, 172717, 172741, 173309, 173501, 192191, 192323, 192373, .

The thirteenth sequence is the other half of the equation which is represented by the cycles and that is the orders. However, in this case the evens out number the odds generally by a ratio of 2 to 1. There-fore, in this race the line Y=2X is employed. What follows is when the lead switches and it is as follows: 3, 53, 61, 67, 137, 173, 181, 191, 197, 199, 223, 227, 233, 239, 251, 283, 419, 439, 463, 467, 503, 563, 571, 599, 607, 613, 619, 631, 659, 787, 1069, 5521, 5531, 10163, 10181, 10271, 10301, 11483, 11491, 22133, 22571, 22573, 22739, 22907, 22937, 23311, 23327, 28151, 28211, 28319, 28517, 28631, 28663, 28759,

28789, 28879, 28909, 29959, 30011, 30323, 32789, 32797, 49459, 49667, 49681, 49843, 50461, 50683, 50909, 50923, 51001, 51031, 55073, 55079, 55109, 55117, 55291, 55609, 55621, 55631, 55691, 55763, 55829, 55837, 55933, 56009, 56113, 56123, 56149, 56333, 60649, 60773, 60793, 61643, 61657, 61667, 61681, 61757, 61813, 61843, 61967, 61991, 62137, 62351, 62401, 62483, 62501, 62507, 62539, 62591, 63179, 63197, 63277, 63467, 63487, 63493, 63521, 64483, 64499, 65267, 65287, 65293, 65393, 66919, 67129, 68489, 68539, 76511, 76579, 76597, 77167, 77213, 79621, 79627, 79687, 80239, 80263, 80363, 80449, 80471, 80489, 81283, 81299, 81307, 81343, 81559, 81629, 81637, 81649, 81667, 82219, 82237, 82699, 82763, 82939, 82963, 82997, 83003, 83023, 83641, 84061, 84067, 84121, 84347, 84377, 84437, 84503, 84523, 86183, 87151, 87181, 87187, 87221, 87293, 87313, 87317, 91373, 91397, 91423, 91757, 91781, 91801, 91811, 92111, 92297, 92311, 94291, 94307, 94321, 94427, 94463, 94477, 94541, 94559, 94811, 94999, 98297, 98323, 98411, 98507, 98737,

100403, 100417, 100519, 129539, 129707, 130241, 130307, 130343, 130649, 130657, 130681, 130769, 131321, 132893, 132911, 132961, 133853, 133993, 133999, 134059, 136133, 136193, 137437, 156979, 157649, 159617, 159667, 160091, 160243, 160313, 160319, 163859, 163883, 163927, 163973, 163981, 163987, 175897, 189347, 189353, 189559, 189593, 189599, 189617, 189653, 189713, 190031, 190093, 190283, 190313, 190403, .

The fourteenth sequence is when they are "neck and neck" and it is as follows: 2, 43, 59, 131, 163, 179, 193, 211, 229, 241, 277, 293, 389, 409, 431, 461, 499, 547, 569, 587, 601, 617, 641, 653, 757, 773, 823, 881, 1063, 5527, 5563, 5639, 10159, 10177, 10267, 10289, 11489, 11587, 22123, 22259, 22567, 22637, 22709, 22727, 22751, 22877, 22921, 23053, 23293, 23321, 27437, 27479, 27919, 27947, 28031, 28201, 28283, 28307,

28349, 28477, 28513, 28559, 28603, 28661, 28751, 28771, 28901, 29483, 29917, 29947, 29989, 30319, 32441, 32783, 32801, 49451, 49639, 49669, 49771, 49801, 50051, 50101, 50459, 50627, 50671, 50893, 50929, 50993, 51061, 51131, 55061, 55103, 55127, 55229, 55259, 55619, 55633, 55663, 55681, 55711, 55733, 55823, 55903, 55931, 56003, 56101, 56131, 56239, 56267, 56311, 60623, 60647, 60779, 61651, 61673, 61781, 61837, 61961, 62057, 62099, 62131, 62213, 62383, 62497, 62533, 62563, 63149, 63247, 63317, 63347, 63443, 63473, 63499, 63599, 66489, 65239, 65269, 65309, 65353, 65381, 67073, 67121, 68449, 68483, 68501, 68531, 76507, 76561, 76829, 77153, 77191, 79613, 79669, 79693, 80231, 80251, 80279, 80317, 80347, 80447, 80473, 81203, 81239, 81293, 81331, 81551, 81619, 81647, 82217, 82231, 82261, 82657, 82723, 82781, 82913, 82981, 83009, 83077, 83597, 83701, 84059, 84089, 84349, 84391, 84463, 84499, 84521, 85621, 86179, 86243, 87133, 87179, 87211, 87277, 87299, 91369, 91387, 91411, 91733, 91771, 91807, 91867, 92003, 92189, 92221, 92237, 92269, 92317, 94229, 94273, 94309, 94399, 94433, 94447, 94531, 94547, 94793, 94961, 98207, 98269, 98317, 98327, 98337, 98407, 98479, 98561, 98731,

100391, 100411, 100517, 129533, 129671, 130223, 130279, 130337, 130363, 130651, 130687, 130729, 130787, 131449, 132887, 132929, 132953, 133843, 133877, 133981, 134033, 134053, 136139, 136189, 136559, 136603, 136999, 156971, 157793, 157823, 157841, 159589, 160073, 160087, 160163, 160201, 160231, 160309, 160343, 163853, 163871, 163909, 163979, 175891, 175919, 189311, 189349, 189583, 189613, 189643, 189701, 189961, 190027, 190063, 190159, 190243, 190271, 190301, 190391,   .

This brings up the point that the "race" of the primes of the two forms 4*k+1 is not cited in your Handbook. This sequence, the fifteenth, then is when the "crossovers" take place and is as follows: 3, 26861, 26879, 616841, 617039, 617269, 617471, 617521, 617587, 617689, 617723, 622813, 623387, 623401, 623851, 623933, 624031, 624097, 624191, 624241, 624259, 626929, 626963, 627353, 627391, 627449, 627511, 627733, 627919, 628013, 628427, 628937, 629371, 629429, 629491, 628513, 629767, 630737, 630827, 632813, 632843, 632897, 632923, 633013, 633599, 633649, 633751, 633797, 633803,

12306137, 12306551, 12308113, 12308587, 12309893, 12309931, 12309961, 12311347, 12311401, 12311443, 12311657, 12311683, 12311837, 12311851, 12311869, 12312043, 12312197, 12312467, 12312497, 12312647, 12312661, 12312787, 12313309, 12314123, 12314321, 12316991, 12317177, 12318107, 12318221, 12318247, 12319753, 12319831, 12321149, 12321187, 12322001, 12322027, 12322217, 12322699, 12323357, 12323383, 12323693, 12324307, 12324353, 12324731, 12324769, 12324839, 12325057, 12325151, 12327857, 12327923, 12327949, 12328079, 12328181, 12328259, 12332989, 12333187, 12334841, 12335003, 12335093, 12335119, 12350017, 12351407, 12360421, 12360643, 12360889, 12360911, 12360937, 12361483, 12361513, 12377311, 12377329, 12377371, 12380293, 12380527, 12382313, 12382367,

951784481, 951784571, 951787157, 951787547, 951787561, 951788179, 951788317, 951789263, 951789989, 951790127, 951790253, 951790327, 951790529, 951790583, 951790633, 951790711, 951791353, 951791527, 951791573, 951791639, 951791669, 951791779, 951791821, 951791887, 951793741, 951793831, 951796141, 951796151, 951796369, 951796567, 951796913, 951796931, 951796981, 951798671, 951798773, 951798803, 951798853, 951798943, 951804209, 951804743, 951804797, 951804859, 951804937, 951804991, 951806341, 951806351, 951806393, 951806963, 951807001, 951807431, 951807473, 951807503, 951807533, 951808719, 951807893, 951807991, 951808673, 951808811, 951809809, 951809939, 951809977, 951811163, 951811801, 951811823, 951811901, 951837659, 951839293, 951839543, 951840521, 951840919, 951841481, 951845347, 951845933, 951846079, 951846433, 951888299, 951890789, 951890839, 951891049, 951891071, 951891181, 951891439, 951891553, 951891667,

951893381, 951893659, 951893809, 951895223, 951895621, 951895751, 951895757,
951895783, 951895937, 951896287, 951896357, 951896591, 951902141, 951902239,
951902333, 951904139, 951904357, 951904391, 951904601, 951904619, 951904669,
951905099, 951905813, 951906323, 951906841, 951907091, 951907097, 951912271,
951912601, 951913507, 951914081, 951914123, 951914741, 951914807, 951914933,
951922219, 951922481, 951922627, 951922681, 951922711, 951922861, 951922967,
951923033, 951923083, 951923153, 951923419, 951923437, 951923447, 951923537,
951923911, 951925033, 951925243, 951925301, 951925483, 951925789, 951932431,
951932521, 951932587, ..., ?, ..., 952223491,
6309280709, ..., ?, ..., 6403150199, 18465126293, ..., ?, ...,
19033524539, .

Discrete races can to some point have a tie. In the race of the primes of the form 4*k1; ie, 4,1(p) = 4,3(p). Here is the sixteenth sequence when P equals: 2, 5, 17, 41, 461, 26833, 26849, 26863, 26881, 26893, 26921, 616769,
616793, 616829, 616843, 616871, 617027, 617257, 617363, 617387, 617411,
617447, 617467, 617473, 617509, 617531, 617579, 617681, 617707, 617719,
618437, 618521, 618593, 618637, 622793, 623171, 623303, 623327, 623351,
623383, 623393, 623431, 623839, 623893, 623929,
623947, 623963, 623983, 624007, 624037, 624049, 624089, 624119, 624139,
624163, 624233, 624251, 626921, 626947, 626959, 627041, 627349, 627383,
627433, 627479, 627491, 627541, 627721, 627811, 627859, 627911, 627973,
628423, 628921, 628939, 629339, 629351, 629381, 629401, 629417, 629483,
629509, 629747, 629773, 630713, 630733, 630823, 630841, 632221, 632561,
632609, 632677, 632717, 632777, 632839, 632881, 632911, 633001, 633091,
633151, 633187, 633427, 633467, 633583, 633613, 663629, 633739, 633757,
633781, 633793, 633799, 633877,
12306061, 12306139, 12306227, 12306247, 12306499, 12306523, 12308069, 12308089,
12308563, 12308579, 12309889, 12309907, 12309953, 12310031, 12310223, 12311311,
12311339, 12311389, 12311419, 12311501, 12311513, 12311653, 12311671, 12311821,
12311839, 12311857, 12312031, 12312101, 12312109, 12312173, 12312193, 12312439,
12312463, 12312493, 12312511, 12312571, 12312631, 12312649, 12312683, 12312767,
12313297, 12313319, 12314083, 12314153, 12314173, 12314293, 12314327, 12314507,
12316847, 12316859, 12316943, 12316987, 12317161, 12318091, 12318109, 12318193,
12318209, 12318227, 12319729, 12319763, 12319787, 12319799, 12319841, 12320017,
12320117, 12320141, 12320201, 12321061, 12321121, 12321137, 12321151, 12321163,
12321193, 12321989, 12322019, 12322169, 12322213, 12322223, 12322243, 12322643,
12322691, 12323237, 12323293, 12323321, 12323371, 12323621, 12323681, 12323711,
12324223, 12324287, 12324341, 12324691, 12324733, 12324761, 12324799, 12324827,
12324841, 12324889, 12324937, 12325037, 12325063, 12325127, 12325147, 12327529,
12327569, 12327617, 12327629, 12327781, 12327841, 12327899, 12327911, 12327937,
12328007, 12328051, 12328133, 12328157, 12328177, 12328243, 12328273, 12329957,
12330061, 12332953, 12332981, 12333127, 12333179, 12333193, 12334837, 12334867,
12334991, 12335021, 12335033, 12335077, 12335111, 12349721, 12349801, 12349993,
12351379, 12360353, 12360401, 12360631, 12360653, 12360877, 12360899, 12360917,
12361411, 12361463, 12361493, 12361651, 12361927, 12377291, 12377317, 12377363,
12377441, 12380281, 12380507, 12380519, 12380597, 12380617, 12380629, 12380653,
12382157, 12382241, 12382289, 12382327, 12382429, 12382637, 12382649, 12382709,
12382757, 12382781, 12423457, 12423613, 12424001, .

This also brings up the "race" of the primes of the two forms 6*k1. This sequence then is as follows: 5, and no other reversal point less than one trillion.

Also the reversals of the primes of the two forms 3*k1 and it is as follows:
2, 608981813029, 608981813357, 608981813707, 608981813717, 608981819119,
608981819273, 608981819437, 608981820869, 608981836423, 608981836481, 608981838529,
608981838617, 608981839633, 608981839727, 608981839891, 608981840939, 608981841109,
608981841659, 608981841733, 608981844953, 608981845009, 608981847101, 608981847343,

608981847371, 608981847469, 608981847611, 608981849659, 608981849747, 608981849803, 608981850161, 608981850511, 608981851421, 608981865589, 608981865599, 608981866177, 608981866739, 608981866837, 608981866931, 608981867227, 608981867261, 608981867287, ..., ?, ..., 610968213797, .

What follows is to establish the relationship, if any, between 3,1(X) and 6,1(X) & 3,2(X) and 6,5(X). The first Prime number being 2 is 2 mod 3 but is not included in the count of Primes modulus 6. The second Prime number being three is not included in either count. Beginning with the third Prime number, that being five, all Primes are of the form 3k1 and 6k1. Furthermore; any Prime 1 mod 3 must be 1 mod 6 and vice versa. The same holds true for Primes 2 mod 3 and 5 mod 6, or if you like, -1 mod 3 and -1 mod 6. Now to establish the relationships stated at the beginning of this paragraph. For all Primes 3,1(X) = 6,1(X) and 3,2(X) = 6,5(X) +1. Remember P1 = two. From this relationship, I have a hard time reconciling the two previous sequences. If indeed 608981813029 is the reversal point for 3 with the Modulus one taking the lead for the first time, backtracking will demonstrate that 608981812919 is a reversal point for 6. Is it the first one? I'm not sure, but 608 billion plus is less than one trillion! Additionally no relationship can be stated for various Primes mod 4 and those of mod 3 or 6.

These previous two series on the "Prime Races" are not very lively so let us move the abscissa or bias the race towards the 1 mod k. But by what mathematical excuse can we employ other than just a desire to make things interesting? Dr. Knuth to the rescue with his convention of letting 1 be the zeroth prime. Now here is what we have when the two sides in the mod 6 "race" are "neck and neck": 5, 11, 17, 23, 31, 41, 47, 67, 73, 83, 97, 103, 109, 127, 157, 167, 211, 227, 233, 241, 379, 439, 1801, 1867, 1873, 1879, . And here is what we have when the two sides in the mod 3 "race" are "neck and neck": 2, 3, 7, 13, 19, 37, 43, 79, 163, 223, 229, . Both of these series were stopped at Primes near 25 million. In view of the lack of entries in these two series, I suggest that they not be used, but I presented them to show their characteristics.

Beginning in the middle of page 33, Stan Wagon discusses "[t]he prime number theorem has an extension that explains the growth of the sequence of primes in the congruence classes modulo some integer. Let n(X,m) be the number of primes px such that pm (mod n). Then the famous theorem of Dirichlet on primes in arithmetic progressions guarantees that each congruence class contains infinitely many primes (provided gcd(m,n) = 1); that is, each function n(X,m) approaches infinity as X approaches infinity. Moreover, when n is prime, then the p-1 classes are uniformly distributed." Therefore; the following series are when a particular congruence class takes the lead and it is represented by the prime number which puts it into the lead.

The seventeenth series is a collection of n(X,m) for n = 5, 7, 8, 9, 10, 11 and 12.

For n =5, the series is as follows: 2, 83, 137, 293, 337, 443, 487, 523, 557, 743, 797, 1213, 1277, 1523, 1657, 1733, 1867, 1973, 2027, 2063, 2797, 2833, 2887, 4733, 5227, 5323, 5437, 5503, 5527, 5623, 5897, 5923, 6007, 6133, 6317, 6353, 6427, 6563, 6607, 6703, 7187, 7283, 7307, 7393, 7477, 8963, 9257, 9323, 9397, 9413, 10037, 10133, 11717, 11863, 11887, 11953, 12007, 12263, 12527, 12743, 13457, 13963, 13997, 14173, 14297, 14303, 14387, 14653, 14887, 16903, 16937, 16963, 17117, 20143, 21487, 21503, 21587, 21713, 21737, 22133, 22807, 23003, 23027, 25913, 26417, 26863, 27367, 31193, 31277, 32633, 36187, 36353, 37567, 37643, 37957, .

For n =7, the series is as follows: 2, 17, 131, 227, 733, 829, 929, 997, 1097, 1123, 1237, 1277, 1447, 1487, 1531, 1627, 1811, 1907, 1993, 2141,

2203, 2267, 2441, 2677, 2707, 3209, 3299, 3433, 3547, 3853, 4003,
4021, 4507, 4679, 4787, 4931, 5081, 5113, 7537, 7577, 7649, 7759, 7817,
8039, 8461, 8543, 8867, 13037, 14327, 14731, 14929, 15263, 15461, 18371,
18913, 18959, 20011, 20051, 20551, 21157, 21481, 21517, 21557, 21577, 21601,
21661, 21727, 21767, 22109, 22189, 23021, 23057, 23371, 23887, 24071, 24097,
24169, 31237, 31267, 31321, 31379, 31531, 32653, 32707, 33521, 33547, 33577,
33617, 33647, 34687, 35983, 45821, 46049, 46769, .

For n =8, the series is as follows: 2, 11, 37, 83, 197, 227, 271, 293, 347,
373, 487, 547, 853, 907, 1069, 1447, 1733, 1831, 1929, 2027, 2053, 2131,
2237, 2251, 2309, 2719, 2749, 3019, 3061, 3083, 3733, 3779,
3877, 3931, 4919, 5179, 5303, 5347, 5407, 6661, 6911, 6949, 6967, 7459,
7789, 11527, 11621, 11887, 12109, 12143, 12157, 13183, 13309, 13339, 13397,
13591, 13613, 14087, 14293, 14327, 14389, 14423, 14741, 14767, 14821, 14887,
14947, 14983, 15077, 15139, 15173, 15227, 15583, 16811, 16831, 17011, 17231,
17683, 17911, 17939, 18199, 18427, 20341, 20507, 20533, 20627, 24077, 24091,
24197, 24659, 24749, 24971, 25037, 25171, 25237, .

For n =9, the series is as follows: 2, 167, 191, 419, 461, 563, 587, 617,
677, 761, 857, 881, 929, 1427, 1451, 1607, 1667, 1777, 1823, 1867, 1913,
2351, 2399, 2459, 4127, 4583, 5039, 5087, 5171, 7283, 7349, 7517, 7547,
76437691, 7901, 8681, 8837, 8933, 11243, 11903, 11927,
18329, 18371, 19913, 19937, 20201, 20369, 23603, 23627, 23981, 24509, 24767,
24943, 24971, 25087, 25169, 25247, 25357, 25391, 26393, 26417, 27743, 27767,
29759, 29837, 30029, 30269, 31249, 31991, 37307, 37571, 38149, 38219, 38891,
38921, 57329, 57809, 58229, 58601, 58679, 58907, 61547, 62129, 63149, 63299,
63689, 63929, 64013, 64037, 64373, 64451, 64553, .

For n =10, the series is as follows: 2, 13, 19, 6173, 6299, 6353, 6389,
16057, 16369, 16427, 16883, 17167, 17203, 17257, 18169, 18517, 18899, 20353,
20369, 20593, 20639, 20693, 20809, 22037, 22109, 22153, 22189, 22343, 22369,
22543, 22679, 23003, 23039, 25147, 25189, 27043,
27329, 27407, 27809, 27827, 28439, 28477, 29009, 29027, 30169, 30197, 30269,
30367, 30539, 30637, 31039, 31397, 32443, 32497, 32573, 35149, 35393, 35509,
35543, 35899, 35933, 36217, 36343, 37987, 38113, 38237, 38653, 38767, 38803,
48109, 48533, 48799, 50053, 52067, 52223, 52757, 55109, 55127, 55399, 58067,
58109, 58757, 59159, 59197, 59239, 59417, 62773, 63067, 63113, 63197, 65203,
65287, 65323, 65497, 66943, 67607, 67733, 68597, 68683, 68777, 68863, 68899,
68947, 69029, 69203, 69857, 70019, 74897, 74959, 75437, 75679, 75797, 76079,
76757, 77929, 78367, 79319, 79367, 79579, 79817, 81929, 81967, 82189, 82217,
82559, 83177, 86729, 86837, 86959, 87427, 87649, 87917, 88169, 88237, 88289,
88337, 88379, 88427, 88499, 88667, 88799, 88817, 89849, 89897, 89909, 90197,
90379, 90407, 90499, 90887, 91199, 91457, 91499, 91703, 92957, 93133, 93377,
93553, 93637, 93703, 93887, 93913, 94007, 94273, 94327, 94543, 96857, 96973,
97157, 97523, 97787, 97843, 98017, 100733, 100787, 101603, 101987, 102023,
102397, 102793, 102877, 102953, 103087, 103123, 103237, 103613, 107867,
.

I have run this series out to 25 million and at no time does a prime 1 mod 10 have the lead. However; this group is not far behind as demonstrated by the following statistics. At the Prime number 24,999,987; which is the 1565927th prime, those primes congruent to one total 391,329, congruent to three total 391,535, congruent to seven total 391,633, and those congruent to nine total 391,429, for a grand total of 1,565,926 prime numbers. Taking into account the Primes 2 and 5, and the two figures are reconciled, save one. Although 1 mod 10 has never had the lead in this series, it has never been far behind and often has not been in last place.

For n =11, the series is as follows: 2, 73, 101, 149, 233, 359, 431, 509,
563, 1051, 1091, 1151, 1259, 1459, 1553, 1811, 2609, 2713, 2741, 4363, 4507,
4561, 4919, 5023, 5189, 6761, 7321, 7433, 7717, 7829, 8039, 8081, 8951,

```
  9043, 9203, 9337, 9851, 9931, 10181, 10457, 11437,
11491, 13099, 19841, 19919, 21379, 21767, 21863, 23531, 23623, 32381, 32423,
  35089, 37573, 37663, 38299, 48131, 48397, 48593, 48677, 49057, 49451, 49741,
  50069, 50159, 50221, 50951, 50993, 51479, 51631, 51659, 51941, 52289, 52489,
  52883, 52973, 53591, 53633, 54277, 54403, 54541, 54779, 57163, 57331, 57493,
  57829, 57881, 59693, 59729, 60859, 60961, 61261,    .
For n =12, the series is as follows: 2, 17, 79, 101, 163, 197, 211, 263,
  281, 379, 401, 443, 461, 479, 631, 677, 739, 809, 907, 953, 1087, 1109,
  1171, 1193, 1543, 1607, 1721, 1759, 2063, 2203, 2417, 2543, 2633, 2711,
  2731, 2753, 3203, 3221, 3323, 3607, 3803, 3847, 3863, 3943, 4397,
4603, 4889, 4999, 5309, 5527, 6869, 6883, 7307, 7853, 8231, 8537, 9103,
  9257, 9883, 10211, 11131, 11171, 11299, 11423, 11551, 12239, 12473, 12491,
  13411, 13463, 13841, 13967, 14009, 14423, 14669, 14771, 15497, 15551, 15581,
  15727, 15761, 15971, 16361, 16787, 17623, 18959, 19231, 19379, 19507, 19997,
  20047, 20249, 20599, 20873, 20959, 22937, 23099, 23201,    .
Primes with even period lengths or order: 7, 11, 13, 17, 19, 23, 29, 47,
  59, 61, 73, 89, 97, 101, 103, 109, 113, 127, 131, 137, 139, 149, 157, 167,
  179, 181, 193, 197, 211, 223, 229, 233, 241, 251, 257, 263, 269, 281, 293,
  313, 331, 337, 349, 353, 367, 373, 379, 383, 389, 401, 409, 419, 421, 433,
  449, 457, 461, 463, 487, 491, 499, 503, 509, 521, 541, 557, 569,    .
Primes with odd period lengths or order: 3, 31, 37, 41, 43, 53, 67, 71,
  79, 83, 107, 151, 163, 173, 191, 199, 227, 239, 271, 277, 283, 307, 311,
  317, 347, 359, 397, 431, 439, 443, 467, 479, 523, 547, 563, 587, 599, 613,
  631, 643, 683, 719, 733, 751, 757, 773, 787, 797, 827, 839, 853, 883, 907,
  911, 919, 947, 991, 1013, 1031, 1039, 1093, 1123, 1151, 1163, 1187,    .
Please notice that at least a cursory review of the two above series would
  give some empirical credibility to William Shanks' theory "that there are
  twice as many primes with even period lengths as there are with odd period
  lengths."  The sixty-fifth entry in the odd list is 521, the 98th prime,
  and in the even list is 1187, the 181st prime.  This ratio of one-third
  to two-thirds has nothing to do with the Artin constant cited earlier.
  In keeping with the above "races," what follows is the prime reversals:
  3, 11, 83, 103, 919, 967, 1523, 1543, 5641, 5651, 5717, 5741, 9293, 9371,
  9403, 9497, 9521, 10501, 10631, 10663, 10733, 10753, 11489, 11497, 10447,
  105983, 106013, 106087, 106163, 106207, 106397, 106417, 107609, 107713,
  107719, 108247, 108677, 108739, 108761, 112337, 112403, 112459, 114067,
  114269, 114281, 114407, 114689,
114713, 114773, 114967, 114997, 115061, 115079, 115337, 115399, 115429,
  115631, 115807, 115853, 115861, 115883, 115901, 116041, 116101, 116933,
  116993, 117053, 118709, 118751, 118927, 119359, 119419, 119591, 119617,
  119809, 120817, 122449, 122819, 122849, 122887, 123083, 123113, 123203,
  123269, 123307, 123593, 123637, 123983, 124133, 124171, 140363, 140411,
  140557, 140663, 140681, 147709, 152147, 152189, 152203, 152419, 152729,
  152767, 154043, 154061, 154081, 154097, 154373, 154571, 154787, 154823,
  154849, 154981, 155003, 155171, 155201, 155299, 155809, 155833, 172399,
  172421, 172489, 172709, 172721, 173531,    .
And the sequence in this race when the two sides are equal: 2, 7, 13, 43,
  53, 71, 79, 101, 107, 809, 911, 941, 953, 1013, 1049, 1493, 1511, 1531,
  5573, 5591, 5639, 5647, 5653, 5693, 5711, 5737, 5849, 9283, 9349, 9397,
  9421, 9433, 9463, 9473, 9491, 9511, 10343, 10499, 10627, 10657, 10667, 10729,
  10739, 10889, 11483, 11491, 22159,
104933, 104953, 104971, 105023, 105167, 105296, 105341, 105389, 105509,
  105527, 105977, 105997, 106019, 106033, 106129, 106181, 106187, 106213,
  106391, 106411, 106427, 106441, 106453, 107603, 107621, 107647, 107687,
  107699, 107717, 107741, 108233, 108293, 108359, 108413, 108439, 108649,
  108727, 108751, 110273, 110291, 112103, 112331, 112397, 112429, 112481,
```

112507, 112559, 112573, 114043, 114073, 114259, 114277, 114299, 114377, 114649, 114679, 114691, 114769, 114781, 114941, 114973, 115057, 115067, 115099, 115309, 115331, 115363, 115421, 115613, 115663, 115793, 115849, 115859, 115879, 115891, 116027, 116047, 116099, 116107, 116359, 116923, 116929, 116953, 116989, 117043, 117109, 117127, 117731, 118691, 118717, 118747, 118903, 118913, 119267, 119293, 119321, 119417, 119569, 119611, 119627, 119653, 119677, 119689, 119759, 119773, 119797, 120383, 120811, 121169, 122443, 122527, 122789, 122827, 122839, 122861, 122869, 122929, 122963, 123001, 123049, 123077, 123091, 123121, 123191, 123229, 123259, 123289, 123583, 123601, 123631, 123953, 123979, 124001, 124067, 124123, 124139, 124153, 124277, 140333, 140351, 140381, 140407, 140521, 140533, 140551, 140617, 140629, 140659, 140677, 147073, 147139, 147661, 147673, 147703, 147853, 147919, 152123, 152183, 152197, 152417, 152723, 152753, 154027, 154057, 154067, 154079, 154087, 154111, 154159, 154279, 154369, 154487, 154501, 154543, 154573, 154613, 154667, 154681, 154747, 154769, 154789, 154807, 154841, 154943, 154991, 155069, 155153, 155167, 155191, 155291, 155723, 155773, 155801, 155821, 155849, 172093, 172169, 172213, 172373, 172411, 172441, 172673, 172687, 172717, 172741, 173309, 173501, 192191, 192323, 192373, .

On page 79, Mr. Beiler begins a sequence and on page 100, Mr. Yates continues it. This sequence is of those Primes and its square that are primitive divisors of the same Repunit and it is as follows: 3, 487, 56598313 (and no others less that 228), . Stated another way, these numbers are the only ones less than 228 that have the same cyclic number.

By relaxing the criteria slightly to "List by ranking of the Repunits, those primes that are factors more than once." This sequence then is as follows: 3, 3, 11, 3, 3, 7, 11, 3, 3, 3, 11, 3, 13, 3, 7, 11, 3, 3, . These occur for Repunits of orders: 9, 18, 22, 27, 36, 42, 44, 45, 54, 63, 66, 72, 78, 81, 84, 88, 90, 99, .

The final sequence (trivial?) represents those denominates which in base 10 terminate; ie, numbers consisting only of some powers of two and/or five: 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100, 125, 128, 160, 200, 250, 256, 320, 400, 500, 512, 625, 640, 800, 1000, 1024, 1250, 1280, 1600, 2000, 2048, 2500, 2560, 3125, 3200, 4000, 4096, 5000, 5120, 6250, 6400, 8000, 8192, 10000, 10240, 12500, 12800, 15625, 16000, 16384, 20000, 20480, 25000, 25600, 31250, 32000, 32768, 40000, 40960, 50000, 51200, 62500, 64000, 65536, 78125, 80000, 81920, 100000, 102400, 125000, 128000, .

Reference: AS1: Abramowitz and Stegun, "Handbook of Mathematical Functions," p864,6-7.

BE3: Albert H. Beiler, Recreations in the Theory of Numbers, Chptr X, "Cycling Towards Infinity," p73-82.

John David Brillhart, Derrick Henry Lehmer, John Lewis Selfridge, Bryant Tuckerman, and Samuel Standfield Wagstaff, Jr., Factorizations of bn 1, b=2,3,5,6,7,10, 11,12 up to high powers, American Mathematical Society, Contemporary Mathematics series, Volume 22, Second Edition, Providence, Rhode Island, 1988.

J.W. Bruce, P.J. Giblin, P.J. Rippon, Microcomputers and Mathematics, pg 29-30, Cambridge University Press, NY, 1990.

KN1: Donald Ervin Knuth, Seminumerical Algorighns, "The Art of Computer Programming," Ed. 2, Vol. 2, Pages 365&6, Addison-Wesley Publ. Co., Reading, Mass, 1980.

C. Stanley Ogilvy & John T. Anderson, Excursions in Number Theory, p60, Dover Publ., N.Y. 1966.

Stephen P. Richards, A Number For Your Thoughts, pg 141-151

& 202-3, New Providence, NJ, 1982.
Hans Riesel, Prime Numbers and Computer Methods for Factorization, Birkhauser, Boston, pg 79-83 & 398-403, 1985.
Joe Roberts, Lure of the Integers, The Mathematical Association of America, pg 283-4, 1992.
Stan Wagon, Mathematica in Action, W. H. Freeman and Co., N.Y., pgs. 33-34, 1991.
David Wells, Dictionary of Curious and Interesting Numbers, p175&219, Penguin, 1986.
Samuel Yates, "Prime Divisors of Repunits," Journal of Recreational Mathematics, v8n1p33-37, Moorestown, NJ, 1975.
Samuel Yates, "Cofactors of Repunits," Journal of Recreati-
v8n2p99-107, Moorestown, NJ, 1975-76. Samuel Yates, "Period Lenghts of Exactly one or two Primes,"
Journal of Recreational Mathematics, v18n1p22-24, Moorestown, NJ, 1985.
Samuel Yates, "Extension of Repunit Table," in the section of Letters to the Editor, Journal of Recreational Mathematics, v n p278-9, 19xx.

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP / CF&GI
RGWv:T4400C
sloane27

26 August 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the following sequences for inclusion in your forth-coming second edition of the above. A Fermat number is of the form 22n +1. For ns between 0 and 4 yeild members which are the only known primes.
"As long ago as 1770 the blind mathematician Euler had proved that if a and b are co-prime, then every factor of a2n + b2n is either the number 2 or of the forn 2n+1 K +1. A Fermat number is a special case of this general theorem where a = 2 and b = 1. Over 100 years later, in 1878, Lucas proved that every prime divisor of 22n +1 must be of the form 2n+2 L +1." [Beiler]
By this, he was able to find the first counter example to Fermat's conjecture, in this case the index is 5. F5 = 4294967297 = 641 * 6700417 = (5*27 +1)(52347*27 +1).
From this the basis for the following sequence is the coefficient "k" which produces the least prime factors of successive Fermat numbers disregarding the first several which are all prime. The sequence begins as follows: 5, 1071, 116503103764643, 1209889024954, 1184, 11131, 39, 7, 82731770, ?(n=14)?>1.279e8 9264, 3150, 59251857, 13, 33629, ?(n=20)?, 534689, ?(n=22)?, 5, ?(n=24)?, 193652, 286330, 282030, ?(n=28)?, 1120049, 149041, ?(n=31)?, 1479, ?(n=33)?, ?(n=34)?, ?(n=35)?, 5, ?(n=37)?, 6, 21, ?(n=40)?, ?(n=41)?, 86970, . If at some future date, I run across an addition, I will forward the same to you.
This series is of the regular Polygons with odd number of sides constructable by straightedge and compass using Euclidian methods alone. It is as follows: 3, 5, 15, 17, 51, 85, 255, 257, 771, 1285, 3855, 4369, 13107, 21845, 65535, 65537, 196611, 327685, 983055, 1114129, 3342387, 5570645, 16711935, 16843009, 50529027, 84215045, 252645135, 286331153, 858993459, 1431655765 and 4294967295. Unfortunately; this series terminates with just the thirty one terms listed above, unless a new prime Fermat number is found. Polygons with sides equal to those above times any integer power of two are also constructable. Therefore; you can build an infinite sequence. It begins as follows: 3, 5, 6, 10, 12, 15, 17, 20, 24, 30, 34, 40, 48, 51, 60, 68, 80, 85, 96, 102, 120, 136, 160, 170, 192, 204, 240, 255, 257, 272, 320, 340, 384, 408, 480, 510, 514, 544, 640, 680, 771, 768, 816, 960, 1020, 1028, 1088, 1280, 1285, 1360, 1536, 1542, 1632, 1920, 2040, 2056, 2176, 2560, 2570, 2720, 3072, 3084, 3264, 3840, 3855, 4080, 4112, 4352, 4369, 5120, 5140, 5440, 6144, 6168, 6528, 7680, 7710, 8160, 8224, 8704, 8738, 10240, 10280, 10880, ...

Reference: BE3: Albert H. Beiler, Recreations in the Theory of Numbers, Chptr XVII, "Round the Perimeter," p173-184.
John David Brillhart, Derrick Henry Lehmer, John Lewis Selfridge, Bryant Tuckerman, and Samuel Standfield Wagstaff, Jr., Factorizations of bn 1, b=2,3,5,6,7,10, 11,12 up to high powers, American Mathematical Society, Contemporary Mathematics series, Volume 22, Second Edition, pgs. lxxxvii-viii, Providence, Rhode Island, 1988.
J.W. Bruce, P.J. Giblin, P.J. Rippon, Microcomputers and Mathematics, pg 153-4, 165, Cambridge University Press, NY, 1990.
KN1: Donald Ervin Knuth, Seminumerical Algorighns, "The Art

of Computer Programming," Ed. 2, Vol. 2, Pages 13, 371, 375 & 380, Addison-Wesley Publ. Co., Reading, Mass, 1980.

Oystein Ore, Number Theory and Its History, p74, Dover Publ, New York, 1988.

Paulo Ribenboim, The Little Book of BIG Primes, p61, Springer-Verlag, New York, 1991.

Hans Riesel, Prime Numbers and Computer Methods for Factorization, Birkhauser, Boston, pg 377-380, 1985.

Joe Roberts, Lure of the Integers, The Mathematical Association of America, pg 214-5, 1992.

Manfred R. Schroeder, Number Theory in Science and Communication, p37-39, Vol.7, Springer-Verlag, Berlin 1984.

David Wells, Dictionary of Curious and Interesting Numbers, p148-150, Penguin, 1986.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane28

15 March 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005
Subject: A Handbook of Integer Sequences
Dear Dr. Sloane,
Please consider the updated sequence for inclusion in your forth-coming second edition of the above. This sequence is a most important one and it is the Harmonic series, SSN 1385.
The sequence begins as follows: 1, 4, 11, 31, 83, 227, 616, 1674, 4550, 12367, 33617, 91380, 248397, 675214, 1835421, 4989191, 13562027, 36865412, 100210581, 272400600, 740461601, 2012783315, 5471312310, 14872568831, 40427833596, 109894245429, 298723530401, 812014744422, 2207284924203, 6000022499693, 16309752131262, 44334502845080, 120513673457548, 327590128640500, 890482293866031, 2420581837980561, 6579823624480555, 17885814992891026, 48618685882356024, 132159290357566703, 359246197441016284, 976532410446924923, 2654490306219185926, 7215652763216299614, 19614177786721165095, 53316863057809197640, 144930260000482107388, 393961292153155329065, 1070897821576167177938, 2911002088526872100231, ...
.

Reference: Journal of Recreational Mathimatics, v10n2p124, 1977.
Mathimatical Mag. v65n5p308, Dec 1992.
Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:hp110+

30 April 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Several months ago, I sent you a letter concerning the primes which surrounded n factorial. This letter concerns the primes which surround p "primorial."

Definition and notation. If n factorial means the product of all the integers from 1 to n, then let us call the product of all the primes form 2 to p, p "primorial." Also if n! denotes n factorial, then let n? denoted n primorial (p is the greatest p < n). I chose the question mark "?" because appreence wise it resembles the exclamation mark "!" the closest and the bend in the question mark is reminisent of the letter "P" for primorial - although I recall vaguely that Dunbar uses #p.

This then will produce our first series and that is p?: 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870, 6469693230, 200560490130, 420738134810, 304250263527210, 13082761331670030, 61488, 782588491410, 32589158477190044730, 1922760350154212639070, 117288381359406970983270, 7858321551080267055879090, 557940830126698960967415390, 40729680599249024150621323470, 32176447673406729078990845
267064515689275851355624017992-790, 23768741896345550770650537601358310, 230556796394551842475310214733175-6070, 23286236435849736090006331688050736307‍0,
2398482352892522817270652163-8692258396210, 25663761175949994144795978153400716483944‍70
279734996817854-9361782761618720678096749972‍30, 31610054640417607788145206291543662493‍2
4014476939333036189094441199026045136645885247730, 525896479052627740-7713717970724119
72047817630210000485677936198920432067‍38-3702541010310, 100146466505991900675092331316‍4
14921823509392793200588757366158410685475838633268645304‍10, 22531953499183117732889023‍6
35375166993717494840635767087951744212057570647889977422429870, 57661522199759516590236
962947420735983927056946215901134429196419130606213075415963491270, 16658990378732521938

... .

p	Last Prime	before p?	the Diff.		Nbr of Primes < p?
1	0	0	0		
2	0	0	0		
3	5	1	3		
5	113	7	30		
7	5,039	1	675		
11	39,916,787		13	2,428,956	
13	6,227,020,777		23	289,620,751	

n		Last Prime before p?	the Diff.
17	355,687,428,095,941	59	
19	121,645,100,408,831,899	101	
23	25,852,016,738,884,976,639,911	89	

From the above table, it is easily discernable that the prime quoted in column two is the nth prime as noted is column four. Therefore; for n >0 the following sequence for (p?) [KN1] is : 0, 0, 1, 3, 9, 30, 128, 675, 4231, 30969, 258689, 2428956, 25306287, 289620751, 3610490805, ~48686917622, ~706003775292, ~10953618190634, ~181035031636349, ~3175094502815204, ~58893-60170794762
... . The tilde "~" means about and is not necessarily exact! and the hyphen "-" means the number is seperated at that point and continues of the next line, but it is still one number.

Two of the above sequences grow so quickly that they will exceed the two lines given in your forthcoming tome. However, the sequence of the differences between p? and its immediate preceeding prime is restated for index n and continued here: 0, 0, 1, 1, 7, 1, 1, 31, 13, 11, 13, 1, 23, 1, 47, 53, 59,

```
  41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 1, 61, 1, 1, 193, 113,
  127, 97, 1, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67,
  59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251,
  103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353,
  463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 1, 109, 163, 311, 173,
  571, 271, 479, ... .
In the preceeding series, there are instances where p?-1 is a prime.  The
  following is that sequence for the index n: 3, 4, 6, 7, 12, 14, 30, 32,
  33, 38, 94, 166, 324, 379, 469, 546, 974, and no others less than 1156.
Not only is the result of the first difference produce a prime, but so is
  the result of the difference between the p? and the second preceeding prime,
  beginning with n =3.  They are as follows: 3, 5, 11, 11, 17, 37, 17, 17,
  17, 13, 61, 17, 59, 71, 61, 43, 113, 71, 41, 101, 191, 103, 191, 179, 71,
  127, 37, 79, 113, 163, 47, 373, 293, 157, 149, 79, 167, 211, 151, 89, 131,
  113, 73, 107, 179, 227, 173, 113, 257, 239, 151, 227, 163, 509, 293, 347,
  643, 373, 457, 109, 199, 661, 317, 659, 277, 547, 197, 139, 887, 211, 953,
  499, 223, 353, 569, 347, 739, 107, 337, 409, 1051, 557, 787, 443, 953, 331,
  281, 103, 1031, 467, 409, 449, 127, 613, 467, 607, 619, 839, 631, ... .
So is the result of the difference between the p? and the third preceeding
  prime also produces a prime, beginning with n =4.  They are as follows:
  7, 13, 19, 19, 43, 29, 23, 43, 17, 67, 43, 71, 89, 239, 47, 197, 151, 43,
  139, 197, 191, 239, 191, 173, 197, 47, 97, 223, 373, 71, 439, 307, 263,
  157, 241, 199, 233, 337, 131, 179, 149, 113, 227, 269, 409, 197, 193, 379,
  271, 181, 419, 367, 701, 751, 379, 811, 401, 463, 263, 839, 773, 683, 811,
  983, 571, 359, 293, 1039, 281, 1399, 523, 271, 523, 877, 461, 743, 109,
  491, 433, 1229, 991, 1987, 661, 1307, 887, 683, 139, 1213, 487, 557, 479,
  179, 719, 829, 1103, 1901, 1427, 659, ... .
What follows is a table of p? until the difference of the previous prime
  becomes composite.                                              The nth term is compos:
 1! :
 2! :
 3! : 1, 3, 4=2*2
 4! : 1, 5, 7, 11, 13, 17, 19, 21=3*7
 5! : 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 49=7*7
 6! : 1, 11, 19, 29, 37, 43, 47, 59, 61, 67, 73, 77=7*7                      12
 7! :       1, 17, 19, 29, 31, 37, 41, 47, 53, 67, 71, 73, 83, 89, 97,
   103, 107, 109, 121=11*11
 8! : 31, 37, 43, 67, 79, 83, 89, 107, 127, 131, 143=11*13             11
 9! : 13, 17, 29, 79, 121=11*11
10! : 11, 17, 23, 37, 41, 53, 89, 103, 121=11*11                              9
11! : 13, 17, 43, 73, 83, 89, 97, 113, 149, 151, 163, 167, 199,
   247=13*19
12! : 1, 13, 17, 37, 61, 73, 83, 107, 139, 169=13*13                         10
13! : 23, 61, 67, 89, 101, 103, 109, 163, 191, 193, 223, 227, 233,
   269, 281, 283, 289=17*17
14! : 1, 17, 43, 47, 61, 67, 89, 131, 197, 199, 211, 227, 233, 313,
   347,    353, 359, 373, 419, 449, 461, 499, 601, 607, 659, 667=23*29      26
15! : 47, 59, 71, 79, 89, 97, 139, 157, 163, 167, 227, 251, 257, 293,
   313, 317, 347, 367, 401, 419, 449, 461, 463, 467, 493=17*29           25
16! : 53, 71, 89, 107, 139, 163, 181, 227, 239, 263, 307, 317, 337,
   353, 359, 367, 421, 431, 433, 449, 457, 467, 563, 569, 571, 587,
   601, 641, 673, 677, 701, 739, 743, 757, 787, 911, 947, 967, 983,
   1007=19*53
17! : 59, 61, 239, 281, 307, 367, 373, 389, 401, 487, 491, 547, 587,
   619, 631, 653, 673, 719, 743, 751, 811, 821, 823, 827, 839, 859,
   883, 941, 947, 1009, 1151, 1163, 1201, 1271=31*41                          34
```

18! : 41, 43, 47, 61, 101, 109, 131, 139, 157, 193, 257, 283, 311,
317, 331, 349, 421, 431, 439, 457, 499, 547, 557, 557, 589=19*31 25
19! : 101, 113, 197, 199, 251, 263, 307, 331, 353, 359, 409, 541,
659, 887, 929, 961=31*31
20! : 31, 71, 151, 163, 179, 191, 251, 257, 269, 353, 367, 397, 433,
443, 521, 601, 613, 631, 641, 659, 661, 797, 823, 853, 857, 859,
863, 877, 1019, 1063, 1097, 1109, 1171, 1181, 1213, 1229, 1279,
1291, 1297, 1361, 1423, 1427, 1493, 1537=29*53 44

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 0, 0, 3, 8, 11, 12, 19, 11, 5, 9, 14, 10, 17, 26, 25, 41, 34, 25, 16, 44, 28, 33, 25, 22, 13, 18, 29, 23, 22, 22, 21, 36, 24, 24, 22, 26, 19, 21, 35, 19, 32, 30, 38, 32, 27, 23, 30, 37, 55, 23, 35, 33, 34, 55, 39, 46, 24, 39, 24, 19, 39, 26, 35, 28, 37, 27, 20, 44, 27, 44, 46, 44, 25, 32, 50, 87, 39, 46, 46, 31, 23, 32, 25, 41, 41, 42, 35, 46, 29, 49, 49, 30, 67, 42, 44, 30, 38, 40, 22, 48, 40, ...

Just like the preceeding sequences are on the side less that the factorial of n, so do we have sequences on the greater side of p? as well.

The mirror image of the first such sequence is the difference between n factorial and the first prime that exceeds it. Keep in mind that 0!=1 by definition (Graham p.111) so this sequence begins with the index n =0. It is as follows: 1, 1, 1, 5, 7, 7, 11, 23, 17, 11, 1, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 1, 67, 223, 107, 127, 79, 37, 97, 61, 131, 1, 43, 97, 53, 1, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 1, 179, 229, 113, 1, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, ...

In the preceeding series, there are instances where p?+1 is a prime. The following is that sequence for the index n: 1, 2, 3, 11, 27, 37, 41, 73, 77, 116, 154, 320, 340, 399, 427, 872, 1477, and no others less than 2043.

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One, the above sequence (the amount by which the first prime number after p? exceeds p?) contains just primes! These series are somewhat analogous to the "Fortunate" numbers, but I shall leave that sequence to a separate letter.

Here is the one which is analogous to the "Fortunate" numbers using the factorials versus the "primorials:" 2, 3, 5, 5, 7, 7, 11, 23, 17, 11, 17, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 47, 67, 223, 107, 127, 79, 37, 97, 61, 131, 311, 43, 97, 53, 61, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 79, 179, 229, 113, 149, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, ...

The preceeding sequence represents the difference between p?+1 and the first prime exceeding p?+1. This one is its mirror image; ie, the difference between p?-1 and the first prime preceeding p?-1 beginning with n =5: 7, 11, 17, 31, 13, 11, 13, 13, 23, 17, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 79, 61, 163, 47, 193, 113, 127, 97, 79, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 449, 109, 163, 311, 173, 571, 271, 479, ...

The difference between the two sequences, the first one citing the differences between p? and its immediate preceeding prime and the second one citing the differences between p? and its immediate succeeding prime, is the "prime gap" surrounding the number p? The largest observed difference being about the number 71! and it is equal to 1608, which equals the spread of the pair (71! - 751, 71! + 857). Since the two sequences above contain only odd numbers, the differences are always even, therefore; the following series is the difference divided by two, beginning with n =3: 1, 3, 7, 4, 6, 27, 15, 11, 7, 15, 45, 10, 45, 38, 45, 39, 95, 30, 31, 52, 93, 102, 95, 48, 22, 84, 127, 54, 94, 40, 19, 145, 87, 129, 49, 22, 85, 68, 66, 121, 90, 78, 146, 95, 156, 78, 71, 79, 225, 60, 65, 175, 66, 305, 192, 196, 215, 205, 420, 101, 186, 213, 160, 300, 132, 167, 117, 118, 804, 132, 187, 189, 198, 135, 246, 215, 264, 105, 392, 139, 255, 345, 257, 108, 639, 366, 153, 168, 581, 238, 125, 136, 328, 181, 270, 240, 337, 250, 309, ...

Not only is the result of the first difference above p? produce a prime, but so is the result of the difference between the p? and the second suceeding prime. They are as follows: 2, 3, 5, 7, 11, 13, 19, 31, 23, 19, 17, 43, 73, 41, 149, 41, 53, 61, 109, 37, 37, 71, 109, 193, 97, 173, 47, 101, 229, 163, 241, 83, 139, 103, 83, 577, 311, 47, 269, 61, 61, 107, 97, 89, 379, 149, 269, 83, 137, 167, 281, 89, 79, 443, 229, 157, 179, 563, 389, 277, 827, 281, 433, 151, 383, 1033, 79, 101, 137, 173, 971, 151, 79, 479, 449, 139, 149, 307, 839, 139, 757, 971, 227, 919, 229, 509, 593, 787, 1187, 1069, 251, 167, 193, 479, 557, 239, 257, 499, 109, 439, 233, ...

So is the result of the difference between the p? and the third succeeding prime after p? also produces a prime. They are as follows: 5, 7, 13, 17, 19, 37, 37, 31, 41, 19, 59, 109, 71, 179, 73, 59, 73, 113, 53, 47, 127, 149, 263, 107, 241, 59, 103, 317, 241, 317, 113, 197, 127, 109, 647, 397, 67, 281, 67, 211, 163, 109, 107, 439, 521, 709, 101, 383, 337, 397, 223, 337, 601, 281, 311, 389, 821, 421, 307, 911, 353, 787, 293, 431, 1051, 233, 461, 241, 307, 991, 251, 107, 487, 997, 151, 197, 647, 881, 157, 937, 1697, 487, 971, 311, 541, 1117, 1129, 1249, 1709, 727, 661, 257, 1153, 607, 461, 337, 523, 239, 547, 563, ...

What follows is a table of p? until the difference of p? and the subsequent prime becomes composite.   The nth term is composite

1! : 1, 2, 4=2*2
2! : 1, 3, 5, 9=3*3
3! : 1, 5, 7, 11, 13, 17, 23, 25=5*5
4! : 5, 7, 13, 17, 19, 23, 29, 35=5*7
5! : 7, 11, 17, 19, 29, 31, 37, 43, 47, 53, 59, 61, 71, 73, 77=7*11
  15
6! : 7, 13, 19, 23, 31, 37, 41, 49=7*7
7! : 11, 19, 37, 41, 47, 59, 61, 67, 73, 79, 107, 113, 127, 131, 139, 149, 157, 169=13*13
8! : 23, 31, 37, 41, 67, 103, 107, 109, 113, 139, 151, 163, 167, 173, 179, 187=11*17
9! : 17, 23, 31, 47, 61, 71, 73, 89, 97, 103, 107, 137, 139, 157, 163, 167, 179, 181, 187=11*17
10! : 11, 19, 41, 47, 53, 83, 97, 127, 131, 149, 167, 169=13*13 12
11! : 1, 17, 19, 29, 31, 37, 59, 109, 113, 131, 139, 149, 173, 179, 191, 211, 227, 239, 169=13*13
12! : 29, 43, 59, 83, 101, 137, 167, 191, 193, 197, 221=13*17 11
13! : 67, 73, 109, 139, 157, 181, 199, 289=17*17 8
14! : 19, 41, 71, 79, 103, 127, 167, 193, 229, 353, 361=19*19 11
15! : 43, 149, 179, 223, 227, 361=19*19
16! : 23, 41, 73, 149, 163, 173, 197, 199, 233, 277, 283, 323=17*19 12
17! : 31, 53, 59, 83, 109, 167, 181, 263, 281, 283, 293, 307, 349, 367, 397, 421, 487, 599, 631, 739, 761, 769, 779=19*41 23
18! : 37, 61, 73, 101, 103, 107, 137, 173, 179, 239, 241, 257, 271,

293, 331, 337, 383, 397, 463, 541, 613, 617, 683, 713=23*31 24
19! : 89, 109, 113, 137, 139, 241, 281, 311, 389, 463, 467, 641,
659, 701, 713=23*31
20! : 29, 37, 53, 79, 89, 137, 139, 157, 167, 211, 271, 277, 283,
353, 367, 379, 439, 491, 571, 617, 619, 661, 673, 743, 839,
919, 941, 953, 997, 1091, 1189=29*41 31

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows:
3, 4, 8, 8, 15, 8, 18, 16, 19, 12, 19, 11, 8, 11, 6, 12, 23, 24, 15, 31,
21, 27, 15, 16, 26, 25, 18, 17, 29, 20, 27, 27, 30, 23, 16, 28, 24, 25,
29, 15, 25, 19, 36, 36, 39, 15, 36, 24, 44, 35, 29, 27, 25, 36, 22, 37,
31, 32, 41, 29, 55, 27, 45, 29, 59, 34, 37, 24, 49, 25, 40, 29, 55, 39,
38, 34, 46, 31, 37, 41, 24, 25, 35, 45, 33, 41, 42, 63, 31, 49, 46, 40,
30, 28, 36, 50, 36, 26, 32, 31, 37, ...

Given the sequence immediately above and its conterpart cited earlier, the sum of the two less the two end points states the number of prime differences surrounding n factorial. It is as follows: 2, 3, 9, 14, 24, 18, 35, 25,
22, 19, 31, 19, 23, 35, 29, 51, 55, 47, 29, 73, 47, 58, 38, 36, 37, 41,
45, 38, 49, 40, 46, 61, 52, 45, 36, 52, 41, 44, 62, 32, 55, 47, 72, 66,
64, 36, 64, 59, 97, 56, 62, 58, 57, 89, 59, 81, 53, 69, 63, 46, 92, 51,
78, 55, 94, 59, 55, 66, 74, 67, 84, 71, 78, 69, 86, 119, 83, 75, 81, 70,
45, 55, 58, 84, 72, 81, 75, 107, 58, 96, 93, 68, 95, 68, 78, 78, 72, 64,
52, 77, 75, ...

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One; for n >2, there exists at least six primes - three below p? and three above p?; for n >9, there exits at least ten primes - five below p? and five above p?; and for n >15, there exits at least twenty primes - ten below p? and ten above p?; which represent the absolute difference of the consecutive primes and p? Additionally, as n grows so does the lower limit. Take for example, n =10, there are 3 consecutive primes immediately below 10!, they being 3628777, 3628783 and 3628789, and 3 consecutive primes immediately above 10!, they being 3628811, 3628819 and 3628841. The differeces are -23, -17, -11, +11, +19 and +41, and these are all prime. This is rather astonishing since a cursory look at Table 1 on page 390 of Knuth [KN1] demonstrates the rarity of this condition.

This sequence is _ p? is as follows: 1, 3, 9, 33, 153, 873, 5913, 46233,
409113, 4037913, 43954713, 522956313, 6749977113, 93928268313, 1401602636313,
2232439252431З, 378011820620313, 6780385526348313, 128425485935180313, 256132749411182
5365226966582126031З, 1177652997443428940313, 27029669736328405580313, 64747807146956 78
1615868811480055382894031З, 419450149241406189412940313, 11308319599659758350180940313,
316196664211373618851684940313, 915795865795107557339530094031З, 274410818470142134209 7
84972494726480649519352666609403 13, 271628086406341595119153278820940313,
89549457052182280906373476801009403 13, 3041877447448223689382559573236 20940-313,
106373357111309672986049072948468209403 13, 382630662501032184766604355-445682020940313,
141463837537273772310825839370265844209403 13, 537169001220-32848899108980803710087562 09
209350510824177718476313715479399982324 2-0940313, 83685033433031550619324264114405589 25
3428937694749412-26143633046945848075576564209403 13, 14392954947003740211575059109390 96
6185455855807420965851263797945309388475855242094031 3, 2720126133346522977702138448994 0
12234234699882671753966529994465178404858813084042094031 3, 562496450681091566738997072 8
264248206017979096310354325882356886646207872272920420940313, 12678163798554051767172 64
62096002783282161263942480669455110881272052560616092042094031 3, 310350532295461996562 5
... . This sequence for all n >4 have a digital root of 9 and for all n >10 are divisible by 99.

This sequence is when the above series less two is prime for the index n:
2, 3, 4, 5, 12, 13, 19, 65, 90, 123, ...

Notice that the lesser significant digits of _ p? become identical for increasing modulus powers of ten as n increases. Therefore; the sequence of the digits in reverse order (least to greatest) is as follows: 3, 1, 3, 0, 4, 9, 0, 2, 4, 0, 2, 9, 8, 2, 5, 6, 3, 3, 2, 4, 4, 6, 5, 5, 2, 5, 0, 9, 3, 0, 5, 0, 1, 3, 9, 5, 3, 2, 3, 4, 0, 8, 4, 9, 9, 7, 0, 1, 1, 2, 6, 8, 3, 7, 4, 8, 6, 8, 7, 4, 9, 7, 4, 7, 4, 2, 2, 9, 0, 0, 4, 3, 3, 0, 5, 6, 5, 8, 6, 5, 0, 0, 2, 6, 6, 5, 1, 5, 9, 7, 8, 8, 1, 6, 2, 0, 2, 8, 1, 2, 1, ...

These last sequences involve the prime factors of _ p?. This one is the first prime factor which exceeds n: 2, 3, 1, 11, 17, 97, 73, 11, 131, 11, 443987, 23, 821, 2789, 107, 225498914387, 163, 19727, 29, 53, 877, 1189548482266089
139, 37, 454823, 107, 431363, 191, 37, 27718264491933-54891007108898387, 1231547, 41, 109, 3072603482270933019578343003268898387, 7523968684626643, 542410073, 379, 127, 956042657, 8453033680104197032254976-173172281742468898387, 1652359939, 1453833833030680829452026172665753916660-6468898387, 134593, 467, 2663, 18701, 2669173798161405013235902281639968551-981897699726468898387, 12890567, 627232351346284457211540208782374857386586-3895011726468898387, 1361, 659, 331, 1301, 2927, composite for n =55 which has a prime factor > 1542319591, ...

And this sequence is when [_ p?] / 99 is a prime for the index n: 2, 4, 5, 6, 7, 11, 16, 22, 30, 34, 40, 42, 47, 49, 68, 74, 79, 168, 202, 245, and for no others less than or equal to 262. Not that this series needs its own recitation, but notice that for all the above primes, the lesser significant digits become identical with increasing modulus powers of ten as n increases, just as the previous series does. Therefore; all that needs to be done is to take the above and divide it by 99.

What follows is not a series but are just observations in the prime gaps.

Reference: W.W. Rouse Ball and H.S.M. Coxeter, Mathematical Recreations and
Essays, 13th edition, Dover Publ., N.Y., p66
JRM v18n2p89.
J.P. Buhler, R.E. Crandall and M.A. Penk, "Primes of the form
n!+1 and 2*3*5***p +1," Math. of Comp v38p639-43, 1982.
Thomas P. Dence, Solving Math Problems in BASIC, p71-2.
Richard K. Guy, Am. Math. Mo. v95n8p699-700&708, Oct 88.
Ronald Lewis Graham, Donald Ervin Knuth, Oren Patashnik, Concrete
Mathematics, A Foundation For Computer Science, Addison-Wesley
Publishing Co., Reading, Mass, 1989.
KN1: Donald Ervin Knuth, Seminumerical Algorithms, "The Art of
Computer Programming," Ed. 2, Vol. 2, Page 365&6, Addison-Wesley
Publ. Co., Reading, Mass, 1980.
Stephen P. Richards, A Number For Your Thoughts, New Providence,
N.J., p70&200.
Joe Roberts, Lure of the Integers, The Mathematical Association
of America, 1992, p62.
S12: Waclaw Sierpinski (1882-1969), A Selection Of Problems in
the Theory Of Numbers, A Pergamon Press Book, the MacMillan Co,.
New York, New York, 1964.
David Wells, The Penguin Dictionary of Curious and Interesting
Numbers, p161, d1986.
Stephen Wolfram "Mathematica," MS-DOS 386/7 Version 2.0.2
released 21 June 1991, the function PrimePi[n!].
Soft Warehouse, "Derive," Version 2.02, 30 Nov 1990,
the function Next_Prime(n!).

Sequentially yours,
Robert G. Wilson v,
Ph.D., ATP/CF&GI

20 September 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forthcoming second edition of the above this fall.

"A number k is called congruent if integers x and y exist such that both x2   ky2 and x2 ky2 are squares. The expression x2   ky2 may also be a negative square as it is always possible to obtain a positive solution from such a negative one. No certain method for ascertaining the congruence of a given number is known even in these modern times when mighty analytic tools are at our disposal. It is also difficult to find a solution even when we know that k is congruent." [Beiler]

The first sequence represents the square free or primitive congruent numbers. They are as follows: 5, 6, 7, 13, 14, 15, 21, 22, 23, 29, 30, 31, 34, 37, 38, 39, 41, 46, 47, 53, 55, 61, 62, 65, 69, 70, 71, 77, 78, 79, 85, 86, 87, 93, 94, 95, 102, . The complete set can be generated by taking the members of the above infinite set and multiplying them by all the integers squared. This will produce the following sequence, although I would advise against its inclusion. It is as follows: 5, 6, 7, 13, 14, 15, 20, 21, 22, 23, 24, 28, 29, 30, 31, 34, 37, 38, 39, 41, 45, 46, 47, 52, 53, 54, 55, 56, 60, 61, 62, 63, 65, 69, 70, 71, 77, 78, 79, 80, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 102, . Instead, might I advice that the non-included members be cited and that they might be entitled the "incongruent" set of integers. "The remaining 49 integers, among which are 1 and 2, are incongruent. Thus x2   y2 = u2 and x2   y2 = v2 are impossible and also x2 2y2 = u2 and x2 2y2 = v2." [Beiler]   They are as follows: 1, 2, 3, 4, 8, 9, 10, 11, 12, 16, 17, 18, 19, 25, 26, 27, 32, 33, 35, 36, 40, 42, 43, 44, 48, 49, 50, 51, 57, 58, 59, 64, 66, 67, 68, 72, 73, 74, 75, 76, 81, 82, 83, 89, 90, 91, 97, 98, 99, 100, 101, .

These numbers are intimately connected with primitive Pythagorean triangles and their generators. These two co-prime integer generators are m and n with m n. Therefore, for increasing ms and ns, the generated primitive congruent numbers which equal mn(m2 n2)/ produce a sequence without repeats, which is as follows: 6, 30, 15, 21, 210, 5, 330, 70, 231, 546, 14, 390, 154, 65, 34, 110, 2730, 3570, 190, 286, 1155, 5610, 1254, 2310, 429, 1785, 1995, 759, 4290, 221, 10374, 8970, 39, 7854, 1330, 1610, 462, 6630, 3135, 4830, 6090, 255, 741, 161, 7, 165, 1131, 465, 9690, 4641, 25806, 7395, 22134, 561, 646, 2990, 4466, 4030, 1190, 13566,
6555, 1482, 5510, 12369, 43890, 1406, 5865, 1365, 1595, 1705, 15015, 9435, 3705, 18354, 357, 15834, 71610, 3885, 4305, 10626, 66990, 8866, 10010, 84570, 72930, 51414, 19866, 966, 1311, 68034, 21390, 98670, 26565, 11914, 6279, 9430, 14835, 2530, 138, 16530, 30030, 31746, 29274, 24510, 6486, 609, 3534, 1122, 1443, 6006, 41, 602, 3102, 6, 78, 52026, 84630, 114114, 15470, 158730, 175890, 19006, 17290, 128310, 1794, 1326, 174, 2139, 3990, 18870, 22386, 1419, 4935, 4134, 651, 4921, 51051, 55965, 58695, 6545, 6251, 41055, 1113, 5394, 957, 140070, 45066, 214890, 60639, 261870, 1885, 29986, 145, 227766, 46110, 124410, 11571, 26970, 178710, 257070, 285090, 311610, 6270, 255990, 51330, 59334, 3255, 6882, 55614, 266910, 75981, 36890, 21855, 806, 86955, 36146, 229710, 38409, 56730, 2046, 1110, 1886, 2470, 23970, 510,

25194, 24486, 798, 1770, 10614, 434, . The cited numbers are for all primitve congruent numbers generated for m & n 25.

Reference: BE3: Albert H. Beiler, Recreations In The Theory Of Numbers, The Queen of
Mathematics Entertains, pgs. 155-7, Dover Publ., NY, 1964.
Oystein Ore, Number Theory and Its History, pgs. 191&202, Dover Publ., NY,
1948.
SI1: Waclaw Sierpinski, Elementary Theory of Numbers, 2nd Ed., Chpt. 2.9., pgs. 62-66, Vol. 31, North-Holland Mathematical Library, El: Science
Publ., NY, 1988.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane31

21 September 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forthcoming second edition of the above. An appropriate title for this sequence might be the Joseph Louis Francois Bertrand sequence. "Each of the primes is less than twice its predecessor in the list." [Hardy] It is as follows:
2, 3, 5, 7, 13, 23, 43, 83, 163, 317, 631, 1259, 2503, 5003, 9973, 19937, 39869, 79699, 159389, 318751, 637499, 1274989, 2549951, 5099893, 10199767, 20399531, 40799041, 81598067, 163196129, 326392249, 652784471, 1305568919, 2611137817, 5222275627, 10444551233, 20889102457, 41778204911, 83556409789, 167112819547, 334225639093, 668451278147, 1336902556279, 2673805112521, 5347610225021, 10695220450027, 21390440900033, 42780881800057, 85561763600057, 171123527200081, 342247054400159, 684494108800091, 1368988217600167, 2737976435200319, 5475952870400627, 10951905740801243, 21903811481602373, 43807622963204729, 87615245926409407, 175230491852818793, 350460983705637557, 700921967411275081, 1401843934822550129, 2803687869645100253, 5607375739290200429, 19913794417256250l3, 3982758883451249999, 7965517766902499957, .

" in 1932, Breusch showed that if n 48, then there exists a prime between n and n. Thus, if 0 k 7 and n 48, then there exits a prime p such that kn + 1 p   (kn + 1)
(k + 1)n.   By the prime number theorem, for every h 1 there exits n0 = n0(h) h such that, if n n0, then there exits a prime p such that n p (1 +) n. [Ribenboim]

For incremented Egyptian Fractions, beginning with h = 1, the series is as follows: 2, 8, 9, 24, 25, 32, 33, 48, 115, 116, 117, 118, 118, 140, 140, 141, 200, 212, 212, 213, 294, 294, 318, 318, 319, 319, 320, 320, 320, 524, 525, 525, 526, 526, 526, 527, 527, 528, 528, 1328, 1329, 1330, 1331, 1331, 1332, 1333, 1333, 1334, 1334, 1335, 1335, 1336, 1336, 1337, 1337, 1338, 1338, 1338, 1339, 1339, 1340, 1340, 1340, 1341, 1341, 1341, 1341, 1342, 1342, 1670, 1670, 1670, 1671, 1671, 1671, 1672, 1672, 1672, 1672, 1673, 1673, 1673, 1673, 1674, 1674, 1674, 1674, 1674, 1952, 1952, 2180, 2180, 2180, 2180, 2181, 2478, 2478, 2478, 2478, 2479, 2479, 2479, 2479, 2980, . As this series increases, the number of repeats does also. Therefore; the table on the following page represents the above series less the repeated entries. An appropriate title might be the J. L. F. Bertrand Reciprocal Postulate Extension. However; to keep track of the growth of this series, the first column is the "h" and the second column is the "p" from the equation cited above.

Reference: William and Fern Ellison, Prime Numbers, p2, John Wiley&Sons, NY, 1985.
Godfrey Harold Hardy & Edward Maitland Wright, An Introduction To The Theory of Numbers, 5th Edition, Sec. 22.3, pgs. 343-5, Clarendon Press Oxford,1979.
Donald Ervin Knuth, The Art Of Computer Programming, 2th Edition, Vol. 1,
"Fundamental Algorithms," p506, Addison-Wesley Publ. Co., Reading, MS, 1973.
Ivan Morton Niven & Herbert Samuel Zuckerman, An Introduction To The

Theory Of Numbers, 4th Edition, Sec. 8.3, pg 224-6, John Wiley&Sons, NY,1980.
Paulo Ribenboim, The Little Book of BIG Primes, pgs. 126, 140, 185-6, Springer-
Verlag, New York, 1991.
Joe Roberts, Lure of the Integers, p101, The Math. Assoc. of Am., Spectrum Series, Vol. ,
SI2: Waclaw Sierpinski, A Selection of Problems in the Theory of Numbers, pgs.
27-8&106, Pergamon Press, Macmillan, NY, 1964.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane30

1	2	115	2974	427	19616	998	58831
2	8	117	3272	431	31397	1015	58832
3	9	122	3273	437	31398	1033	58833
4	24	126	3274	443	31399	1051	58834
5	25	131	3275	449	31400	1070	58835
6	32	137	3276	456	31401	1090	58836
7	33	143	3277	462	31402	1111	58837
8	48	144	4298	469	31403	1119	59281
9	115	149	4299	476	31404	1141	59282
10	116	154	4300	484	31405	1163	59283
11	117	160	4301	491	31406	1186	59284
12	118	162	4832	499	31407	1188	60539
14	140	167	4833	507	31408	1211	60540
16	141	173	4834	515	31409	1236	60541
17	200	175	5592	524	31410	1262	60542
18	212	181	5593	533	31411	1289	60543
20	213	187	5594	541	34061	1317	60544
21	294	192	5750	550	34062	1346	60545
23	318	199	5751	559	34063	1359	79699
25	319	206	5752	568	34064	1375	79700
27	320	213	5754	578	34065	1380	89689
30	524	217	6492	588	34066	1402	89690
31	525	224	6493	598	34067	1424	89691
33	526	231	6918	609	34068	1447	89692
36	527	239	6919	620	34069	1471	89693
38	528	242	7254	631	34070	1495	89694
40	1328	250	8468	643	34071	1521	89695
41	1329	257	8469	648	35617	1547	89696
42	1330	265	8470	660	35618	1574	89697
43	1331	266	9552	673	35619	1602	89698
45	1332	273	9553	674	35677	1631	89699
46	1333	281	9554	687	35678	1652	107377
48	1334	290	9555	700	35679	1678	107378
50	1335	294	9974	711	43331	1705	107379
52	1336	303	9975	723	43332	1732	107380
54	1337	312	9976	735	43333	1761	107381
56	1338	322	9977	748	43334	1790	107382
59	1339	333	9978	751	44293	1810	110359
61	1340	338	10800	764	44294	1840	110360

64	1341	346	11744	778	44295	1871	110361
68	1342	356	11745	792	44296	1903	110362
70	1670	357	15684	806	44297	1937	110363
73	1671	365	15685	821	44298	1950	155928
76	1672	374	15686	836	44299	1974	155929
80	1673	378	19610	852	44300	2000	155930
84	1674	385	19611	869	44301	2026	155931
89	1952	393	19612	887	44302	2052	155932
91	2180	401	19613	900	4589	2080	155933
95	2181	409	19614	918	45894	2108	155934
96	2478			919	48679	2137	155935
100	2479			937	48680	2166	155936
104	2480			955	48681	2197	155937
107	2972			974	48682	2228	155938
111	2973	418	19615	994	48683	2260	155939

18 April 1994

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
600 Mountain Avenue
Murry Hill, New Jersey 07974-0636
908+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

The following sequence should be seriously considered for inclusion in your forthcoming second edition of the above. The sequence is infinite because its Schnirelmann Density is greater than that of the prime number sequence. I am speaking of the sequence of "weird" numbers, defined as those abundant numbers none of whose proper or aliquot divisors or any subset thereof can be arranged such that when added together equal the number. I am only submitting the "primitive" or "generative" weird numbers. This is because once a number is on the list, any prime greater than the sum of all of the divisors multiplied be the generating number is itself a "weird" number.

The sequence begins: 70, 836, 4030, 5830, 7192, 7912, 9272, 10792, 17272, 45356, 73616, 83312, 91388, 113072, 243892, 254012, 338572, 343876, 388076, 519712, 539744, 555616, 682592, 786208, 1188256, 1229152, 1713592, 1901728, 2081824, 2189024, 3963968, 4128448, 4145216, 4199030, 4486208, 4559552, 4632896, 4790812, 4960448, 5440192, 5568448, 6460864, 6621632, 7354304, 7470272, 8000704, 8134208, 8812312, 9928792, 11339816, 11547352, 12872512, 12979264, 13086016, 14528576, 15126992, 21872655, 28279232, , 34869056,
.

Reference: Stan J. Beknoski and Pál Erdös, "PseudoPerfect vs. Weird," Math. of
Computation, v28n126pgs 617-623, April 1974.
Thomas P. Dence, Solving Math Problems in BASIC, pgs 86-7, TAB Books, No. 1564, Blue Ridge Summit, PA 17214, 1983.
Ross Honsberger, Mathematical Gems I, Math. Assoc. of Am., Dociani Math. Expositions, Nbr. 1, p115, Washington, D.C., 1973.
Sidney Kranitz, "A Search for Large Weird Numbers," Journal of Rec. Math. v9n2pgs82-5, 1976-7.
Stephen P. Richards, A Number For Your Thoughts, pgs 83-5, New Providence, NJ 07974, 1982.
David Wells, Dictionary of Curious and Interesting Numbers, p129, Penguin Books, London, 1982.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane33

37307713155613000,
679562217794156938, 14950368791471452636, 301958232385734088196, 724699757725761-8116704,
160755658074834738495566, 4179647109945703200884716, 101019988341178648-636047412,
2828559673553002161809327536, 73990373947612503295166622044, .
A1 = 1, A2n = (2·n(2)·A2 (deleted bad chars here)
The second-order Eulerian Triangle, unlike the first-order Eulerian Triangle, is not symmetrical, and therefore I have labeled the diagonal layers as either left-handed or right-handed. The left-handed series are ones which begin at the top on the right-hand side, and proceed down and to the left; acquiring the majority of their entries from the left side of the triangle. The right-handed series is generated in the opposite direction. The two outer layers, the zeroth-levels, are trivial (1,0,0,0, , or 1,1,1,1, ,) and are "transparent."
First-level right-handed second-order Eulerian Coefficients: 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 47901600, 6227020800, . = n! which is SSN 659.
Second-level right-handed second-order Eulerian Coefficients: 1, 8, 58, 444, 3708, 33984, 341136, 3733920, 44339040, 568356480, 7827719040, 115336085760, 1810992556800, 30196376985600, 532953524275200, 9927928075161600, 194677319705702400, 4008789120-817152000, 86495828444928000000, 1951566265951948800000, 4595893390250072064(
1127742429671124664320000, 28788157126772471070720000, 763382846937681994383360-000, .
Third-level right-handed second-order Eulerian Coefficients: 1, 22, 328, 4400, 58140, 785304, 11026296, 162186912, 2507481216, 40788301824, 697929436800, 12550904017920, 236908271543040, 4687098165573120, 97049168010017280, 20998302094029312(
47405948832458496000, 1115089078488795648000, 27290469545695931904000, .
Fourth-level right-handed second-order Eulerian Coefficients: 1, 52, 1452, 32120, 644020, 12440064, 238904904, 4642163952, 92199790224, 1883079661824, 39689578055808, 865023253219584, 19515249341231616, 455924361142656000, 11030149104146035200, 276260563641659673600, 7159894093909966924800, 191909061666637527(
.
Fifth-level right-handed second-order Eulerian Coefficients: 1, 114, 5610, 195800, 5765500, 155357384, 4002695088, 101180433024, 2549865473424, 64728375139872, 1666424486271456, 43708768764064128, 1171582385481357696, 32157753536587053312, 905080567903692754176, 26142102647955180877824, 5316245463034160902656000,
.
Sixth-level right-handed second-order Eulerian Coefficients: 1, 240, 19950, 1062500, 44765000, 1648384304, 56041398784, 1818188642304, 57494373464592, 1797171220690560, 56071264983487776, 1758073054805500608, 55666251271784164032, 178619858003503868-3136, 58231173019431358391040, 1932284140872498212212224, 6535127482207838404407-2448 .
Seventh-level right-handed second-order Eulerian Coefficients: 1, 494, 67260, 5326160, 314369720, 15548960784, 687720046384, 28299910066112, 1111747472569680, 4243015660-3438560, 1592677516697452416, 59321137058404865280, 2206689692993315764416, 8238-0712138316751438720, 3097746823026022664711040, 117653457298688606007304704,
.
Eighth-level right-handed second-order Eulerian Coefficients: 1, 1004, 218848, 25243904, 2051482776, 134323420224, 7634832149392, 394365587815520, 19076135772884-080, 882128824583603520, 39572673298262064576, 1740743150455672062336, 7568703107-1216250620(
3272566839930489537515520, 141370640703374536558615680, .
First-level left-handed second-order Eulerian Coefficients: 0, 2, 8, 22, 52, 114, 240, 494, 1004, 2026, 4072, 8166, 16356, 32738, 65504, . = 2n+1 2·n 2 = twice the SSN 1382.
Second-level left-handed second-order Eulerian Coefficients: 0, 6, 58, 328, 1452, 5610, 19950, 67260, 218848, 695038, 2170626, 6699696, 20507988, 62407890, 189123286, 571432-036, 1722945672, 5187185766, 15600353130, 46882846680,

140820504700, 422822222266, 1269221639358, 3809241974028, 11431014253872, 34299887862990, 102913890665170, .

Third-level left-handed second-order Eulerian Coefficients: 0, 24, 444, 4400, 32120, 195800, 1062500, 5326160, 25243904, 114876376, 507259276, 2189829808, 9292526920, 38917528600, 161343812980, 663661077072, 2713224461136, 11039636532120, 447513595-47420, 180880752056880, 729437469424920, 2936354055479384, 11803800417328004, .

Fourth-level left-handed second-order Eulerian Coefficients: 0, 120, 3708, 58140, 6440-20, 5765500, 44765000, 314369720, 2051482776, 12669817776, 75016052228, 429826006340, 2400028258540, 13128749622100, 70645406312880, 375127847107776, 1970602091678640, 10261477010081640, 53052688072757580, 272679551198119980, 1394763567175871460, .

Fifth-level left-handed second-order Eulerian Coefficients: 0, 720, 33984, 785304, 1244-0064, 155357384, 1648384304, 15548960784, 134323420224, 1084676512416, 83084443279-68, 61026142132648, 433357644035008, 2994008352873048, 20224703119250448, 13410256-5517167072, 875557068403433472, 5643278536803703152, 35981778743732522112, .

And finally, the series which represents the largest coefficients in the nth row is as follows: 1, 2, 8, 58, 444, 4400, 58140, 785304, 12440064, 238904904, 4642163952, 101180433024, 2549865473424, 64728375139872, 1797171220690560, 56071264983487776, 175807305480-5500608, 59321137058404865280, 2206689692993315764416, 75687031071216250620480, 3272566839930489537515520, 141370640703374536558615680, 6123574537880253792875-345664, .

Incidentally the index of the kth element in the nth row which is the largest is as follows: 0, 1, 1, 2, 3, 3, 4, 5, 5, 6, 7, 7, 8, 9, 9, 10, 11, 11, 12, 12, 13, 14, 15, . This shows that the largest kth element in the nth row approaches an optimum ratio for k/n of about 15/23.

Reference: David Neal, "The Series (n=1 nmxn and a Pascal-like Triangle," The College
Mathematics Journal, v25n2p99-101 & 161, Washington, D.C., March 1994; also see CMJ v24n2p184, March 93.
Ronald Lewis Graham, Donald Ervin Knuth, Orin Patashnik, Concrete Mathematics, pgs 253-8, Addison-Wesley Publ. Co., Reading, Mass., 1989.
E.H. Neville, Designed and Compiled by, Royal Society Mathematical Tables, Volume I, The Farey Series of Order 1025, Displaying Solutions of the Diophantine Equation bx (ay = 1 Published for the Royal Society by th University Press, Cambridge, page xii, 1950.

Sequentially yours,
Robert G. Wilson v
Ph.D., ATP/CF&GI
RGWv:T4400C
sloane34

PS: Yet another triangle is one mentioned by Richard Blecksmith, "Monotonic Numbers," Mathematics Magizine, v66n4np259, Washington, D.C., Oct 93; but I will leave that for another time!