REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
23-06-2017	Final Report	15-Aug-2015 - 14-Feb-2017

4. TITLE AND SUBTITLE	
Final Report: An Instrumentation Request for Upgrading a Mid-Size Heterogeneous Computing System Supporting Research and Educational Activities in Computational Dynamics	5a. CONTRACT NUMBER W911NF-15-1-0386
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 611103
6. AUTHORS Dan Negrut	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Wisconsin - Madison 1.00 RESEARCH & SPONSORED PROGRAMS 21 N. PARK STREET SUITE 6401 MADISON, WI 53715 -1218	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO 11. SPONSOR/MONITOR'S REPORT NUMBER(S) 66876-MA-RIP.1

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

See attached document.

15. SUBJECT TERMS

high performance computing, GPU computing, parallel computing, computational dynamics

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			DAN NEGRUT
UU	UU	UU	UU		19b. TELEPHONE NUMBER 608-890-0914

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Title

Final Report: An Instrumentation Request for Upgrading a Mid-Size Heterogeneous Computing System Supporting Research and Educational Activities in Computational Dynamics

ABSTRACT

See attached document.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

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TOTAL:

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Books

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Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
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Names of Post Doctorates

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Names of Faculty Supported

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Names of Under Graduate students supported

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Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Please see attached document.

Technology Transfer

Technology transfer took place as described in the attached document.

W911NF-15-1-0386–DURIP Final Project Report

An Instrumentation Request for Upgrading a Mid-Size Heterogeneous Computing System Supporting Research and Educational Activities in Computational Dynamics

Dan Negrut

University of Wisconsin – Madison

June 23, 2017

The funds associated with the US Army W911NF-15-1-0386 DURIP grant have been spent as shown in Table 1.

Cost (in $)	Description
59.99	Cables
46 370.00	Inter CPU nodes
10 097.45	Node for GPU cards
6192.27	NVIDIA GTX1080 GPUs to replace obsolete GTX480 GPUs
20 640.90	NVIDIA GTX1080 GPUs for new nodes
8944.39	Supermicro SSE-X3348 10GBase-T RJ45 switch
6200.00	New Euler node
6050.00	New Euler node
8825.00	New Euler node
85 400.00	NVIDIA GTX1080 GPUs nodes and file server with 10 8TB HDDs
198 780.00	TOTAL EXPENSES

Table 1: Project expenses.

The grant funds were used to update an existing heterogeneous CPU/GPU cluster called Euler [1]. The hardware assets are part of a cluster that is hosted in the Mechanical Engineering building in a climate-controlled room (4170ME), which is 70 feet away from the PI office (room 4150ME). The cluster is managed by a system administrator hired by the PI's lab. There were no salary expenses of any type associated with the W911NF-15-1-0386 project; the system administrator has always been paid from a different funding stream associated with the university. The Euler cluster has experienced almost 100% utilization owing to use in research, educational, and project activities.

Research activities. The Euler cluster has been used over the last two years to produce results reported in several journal publications: numerical methods [2, 3, 4, 5]; fluid-solid interaction (FSI) [6, 7, 8]; terramechanics/geomechanics [9, 10, 11]; additive manufacturing [12], and biomechanics [13]. Two more manuscripts that summarize research results in the areas of FSI and geomechanics are under review [14, 15].

We also produced a number of technical reports that are available online [16]. The more relevant ones are: [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39].

Educational/Outreach/Technology Transfer activities. Since 2013, the PI has organized an event tied to the topic of advanced computing and vehicle mobility [40]. The list of event participants includes Open Source Robotics Foundation, Jet Propulsion Laboratory, NASA, Japan Aerospace Exploration Agency, Caterpillar, P&H Mining, MSC.Software, Simertis Gmbh, BAE Systems, Eaton Corporation, Rescale, Red Cedar Technology, NVIDIA, Nevada Automotive Test Center, Harley-Davidson Motor Company, Oshkosh Corporation, John Deere, Statoil, Proctor and Gamble, Cooper Industries, GlaxoSmithKline, Motionport, Altair, Hendrickson, US Army TARDEC, US Army ERDC, Japan Department of Defense, Intuitive Machines, Energid Technologies, SmartUQ, SimLab Soft, Progeneric Systems, Function Bay, Trek Bicycles, D-Wave, Dynamic Simulation Technologies, University of Wisconsin-Madison, Johns Hopkins University, University of Iowa, Colorado State University, Georgia Institute of Technology, University of Parma (Italy), Politecnico di Milano (Italy), MIT, Columbia University, Darmstadt University (Germany), University of Illinois at Chicago, University of Michigan, University of Aarhus (Denmark), Beijing Institute of Technology (China), Fraunhofer Institute for Industrial Mathematics at Kaiserslautern University (Germany), and Inha College (S. Korea). All meetings have been organized at UW-Madison in May and December of each year, with the exception of the last edition, which was organized in Novi, Michigan, in partnership with US Army TARDEC. The purpose of these meetings is to serve as an applied computing idea marketplace, to facilitate technology transfer through strong industry ties, and to promote the simulation software solutions developed by the PI's lab. These meetings are self-supporting as the participants pay registration fees.

Secondly, the cluster has been used for student education purposes. This instrumentation project offered students taking Negrut's graduate course "ME759: High Performance Computing for Engineering Applications" – an offering cross-listed in Mechanical Engineering, Computer Science, Engineering Mechanics and Aeronautics, and Electrical and Computer Engineering [41] – the opportunity to work on a class project related to parallel computing in Science and Engineering. More than 90 students from 10 departments at the University of Wisconsin-Madison registered for this graduate class in Fall 2015, with 66 of them concluding it. The Fall 2015 offering of ME759 was the most popular graduate course in the College of Engineering at the University of Wisconsin–Madison.

Army projects. The PI's group has been involved over the last two years in three large Army project:

- US Army Rapid Innovation Funding Project W56HZV-14-C-0254 [9/1/2014 - 9/1/2016] – "A Physics-Based High Performance Computing Capability for Ground Vehicle Mobility Analysis".
- US Army TARDEC Project W56HZV-08-C-0236 [8/1/2015 - 6/15/2016] – "Development of a High Performance Computing Software Infrastructure for the Modeling and Simulation of Multibody Dynamics Applications: Part 2".
- US Army TARDEC Project W56HZV-08-C-0236 [4/1/2017 - 3/31/2018] – "Development of a High Performance Computing Software Infrastructure for the Modeling and Simulation of Multibody Dynamics Applications: Part 3".

The most important component delivered under these projects is a parallel software infrastructure that supports modeling/simulation/visualization in multi-physics problems. This simulation tool, called Chrono, is designed to solve computational dynamics problems. Chrono's strength lies in its ability to model many-body systems such as those encountered in vehicle-soil interaction. Chrono has a suite of discipline-specific vertical modules: Chrono::Engine, Chrono::FEA, Chrono::FSI, Chrono::Vehicle, and Chrono::Parallel – not all at the same level of maturity. Chrono is used by US Army TARDEC and US Army ERDC (CREATE-GV and Proteus projects). The Army collaborators have been trained in the use of Chrono in a one week tutorial organized by the PI on-site at US Army TARDEC in Warren, Michigan [42]. Some highlights of Chrono are as follows.

- Chrono::Engine is the core module of Chrono and offers simulation support for common rigid multibody dynamics problems. As such it routinely handles rigid bodies, mechanical joints, and frictional contact. The latter comes in two flavors: penalty [43] and complementarity. The complementarity approach, which is anchored in the methodology of [44, 45], leads to a set of differential variational equations solved using the APGD [2] or Barzilai-Borwein [46] methods. Chrono relies on a half-implicit Euler integration method used in the CCP framework proposed in [47], or on a HHT integration scheme [48] used for friction-free rigid and flexible body dynamics. The current formalism in Chrono is Fully Cartesian, descriptor form, as described in [49].
- Chrono::FEA is a module that has been developed to support nonlinear finite element analysis of flexible bodies. There are three sources of nonlinearity in the solution implemented by this module: (a) large displacements and large deformations; (b) nonlinear boundary conditions, such as friction and contact forces; and, (c) nonlinear material laws. The FEA methodology implemented is as follows:
  - ANCF formulation as defined by [50]. Augmented with enhanced assumed strain (EAS) and assumed natural strain (NAS) [51]. Valid for large displacements, large rotations, small strains. Can handle nonlinear material models.
  - Co-rotational approach as described in [52]. Valid for large displacement, large rotations, small strains. Can handle some nonlinear material models.

  Chrono::FEA provides preliminary support for plasticity which comes into play for continuum terrain. A first approach relies on ANCF. A second approach relies on a methodology reported in [53, 54], which uses a meshless method; i.e., the material point method (MPM) to solve the equations of motion.
- Chrono::FSI is a module that provides support for fluid–solid interaction phenomena. For handling the fluid part, Chrono uses the Smoothed Parti-

cle Hydrodynamics approach proposed in [55, 56]. For handling the rigid body dynamics part Chrono uses the half-implicit symplectic Euler integration method [57] that is standard in Chrono. The Fluid-Solid coupling was done using complementarity conditions. The overall problem is posed like one big Cone Complementarity Problem (CCP). This CCP is solved using the APGD method [2] or Barzilai-Borwein method [46].

- Chrono::Vehicle is a module that allows simulation of wheeled and tracked vehicles. It has templates for rapid model generation. This is the module that is most widely used insofar the priorities of the US Army are concerned; i.e., off-road mobility studies.

- Chrono::Parallel supports the simulation of large models with millions of degrees of freedom. Its main purpose is to accelerate core functionality that is provided by Chrono::Engine. Unlike the latter, Chrono::Parallel only supports a subset of services, but it sees to them in a parallel fashion. The list of such services supported includes: collision detection, proximity computation in FSI, handling of friction and contact, function and Jacobian evaluation in Chrono::FEA, etc.

The Euler cluster augmented through this DURIP project is the resource used by the PI's lab to run Chrono. The Chrono software and its verticals are available as open source software accessible in an online public repository [58]. Project main page: [59]. Chrono API: [60]. Approximately 2500 pages of software documentation are available at [61]. More than 30 demos that explain the use of the code are available in the Chrono distribution. Several white papers are available on the Project Chrono website [62]. User questions have been and continue to be answered on the Chrono Forum [63], which as of June 2017 had more than 150 members. Information about daily Chrono metrics (build and test) is available online at [64, 65]. Chrono has been released as open-source under a BSD-3 license, which ensures a very permissive use and distribution mechanism.

References

[1] SBEL. Euler: A CPU/GPU–Heterogeneous Cluster at the Simulation-Based Engineering Laboratory, University of Wisconsin-Madison. `http://sbel.wisc.edu/Resources/`. Accessed: 2015-02-09.

[2] H. Mazhar, T. Heyn, A. Tasora, and D. Negrut. Using Nesterov's method to accelerate multibody dynamics with friction and contact. *ACM Trans. Graph.*, 34(3), 2015.

[3] Radu Serban, Daniel Melanz, Ang Li, Ilinca Stanciulescu, Paramsothy Jayakumar, and Dan Negrut. A GPU-based preconditioned Newton-Krylov solver for flexible multibody dynamics. *International Journal for Numerical Methods in Engineering*, 102:1585–1604, 2015.

[4] Ang Li, Radu Serban, and Dan Negrut. Analysis of a splitting approach for the parallel solution of linear systems on gpu cards. *SIAM Journal on Scientific Computing*, 39(3):C215–C237, 2017.

[5] Daniel Melanz, Luning Fang, Paramsothy Jayakumar, and Dan Negrut. A comparison of numerical methods for solving multibody dynamics problems with frictional contact modeled via differential variational inequalities. *Computer Methods in Applied Mechanics and Engineering*, 320:668 – 693, 2017.

[6] A Pazouki and D Negrut. A numerical study of the effect of particle properties on the radial distribution of suspensions in pipe flow. *Computers and Fluids*, 108:1 – 12, 2015. used to be Pazouki2014a.

[7] Arman Pazouki and Dan Negrut. Numerical investigation of microfluidic sorting of microtissues. *Computers & Mathematics with Applications*, 72(2):251–263, 2016.

[8] W. Hu, W. Pan, M. Rakhsha, Q. Tian, H. Hu, and D. Negrut. A consistent multi-resolution Smoothed Particle Hydrodynamics method. *Computer Methods in Applied Mechanics and Engineering – in press*, page 00, 2017.

[9] J. A. Fleischmann, R. Serban, D. Negrut, and P. Jayakumar. On the importance of displacement history in soft-body contact models. *ASME JCND*, 2015. doi: 10.1115/1.4031197.

[10] Daniel Melanz, Paramsothy Jayakumar, and Dan Negrut. Experimental validation of a differential variational inequality-based approach for handling friction and contact in vehicle/granular-terrain interaction. *Journal of Terramechanics*, 65:1 – 13, 2016.

[11] Antonio Recuero, Radu Serban, Bryan Peterson, Hiroyuki Sugiyama, Paramsothy Jayakumar, and Dan Negrut. A high-fidelity approach for vehicle mobility simulation: Nonlinear finite element tires operating on granular material. *Journal of Terramechanics*, 72:39 – 54, 2017.

[12] Hammad Mazhar, Tim Osswald, and Dan Negrut. On the use of computational multi-body dynamics analysis in SLS-based 3D printing. *Additive Manufacturing*, (http://www.sciencedirect.com/science/article/pii/S2214860416300963), 2016.

[13] R.C. Smith, K.W. Choi, D. Negrut, and Darryl G. Thelen. Efficient Computation of Cartilage Contact Pressures within Dynamic Simulations of Movement. *Computer Methods in Biomechanics and Biomedical Engineering: Imaging and Visualization*, (March), 2016.

[14] A. Pazouki, M. Kwarta, K. Williams, R. Likos. W., Serban, J. Jayakumar, and D. Negrut. Compliant versus rigid contact – a comparison in the context of granular dynamics. *Phys. Rev. E (under review)*, 0:0, 2017.

[15] H. Mazhar, A. Pazouki, M. Rakhsha, P. Jayakumar, and D. Negrut. A differential variational approach for handling fluid-solid interaction problems via Smoothed Particle Hydrodynamics. *Journal of Computational Physics (under review)*, 0:0, 2017.

[16] SBEL. Publications Webpage, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison. `http://sbel.wisc.edu/Publications`, 2014.

[17] A. Recuero, R. Serban, B. Peterson, H. Sugiyama, and D. Negrut. Wheeled Vehicle Mobility Studies in Chrono: Nonlinear Finite Element Tires Operating on Granular Terrain. Technical Report TR-2016-07: `http://sbel.wisc.edu/documents/TR-2016-07.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[18] K. Williams. Cone penetration results for 3mm glass beads and 20-30 Ottawa sand. Technical Report TR-2016-04: `http://sbel.wisc.edu/documents/TR-2016-04.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[19] K. Williams. Direct Shear and PIV Analysis Results for 3mm Glass Beads and 20-30 Ottawa Sand. Technical report, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[20] D. Heller. Investigation of Charm++ SPMV performance using CSR format. Technical Report TR-2015-04, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2015.

[21] D. Melanz, H. Mazhar, and D. Negrut. Using an Anti-Relaxation Step to Improve the Accuracy of the Frictional Contact Solution in a Differential Variational Inequality Framework for the Rigid Body Dynamics Problem. Technical Report TR-2016-02: `http://sbel.wisc.edu/documents/TR-2016-02.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[22] J. Madsen and A. Dirr. Validation of a steady-state magic formula tire in chrono with a commercial software implementation. Technical Report TR-2014-14: `http://sbel.wisc.edu/documents/TR-2014-14.pdf`, University of Wisconsin-Madison Simulation Based Engineering Lab, 2014.

[23] J. Madsen. Validation of a Single Contact Point Tire Model Based on the Transient Pacejka Model in the Open-Source Dynamics Software Chrono. Technical Report TR-2014-16: `http://sbel.wisc.edu/documents/TR-2014-16.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2014.

[24] J. Fleischmann. DEM-PM Contact Model with Multi-Step Tangential Contact Displacement History. Technical Report TR-2015-06, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2015.

[25] D. Melanz, M. Hammad, and D. Negrut. A study of the effect of CCP–relaxation of the equations of motion in a differential variational inequality framework for handling frictional contact. Technical Report TR-2015-10, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2015.

[26] M. Hammad and D. Negrut. Representing fluid dynamics as a many-body dynamics problem: a vehicle fording analysis test case. Technical Report TR-2016-01, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[27] A. Pazouki, B. Song, and D. Negrut. Boundary condition enforcing methods for Smoothed Particle Hydrodynamics. Technical Report TR-2015-08: `http://sbel.wisc.edu/documents/TR-2015-08.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2015.

[28] Michael Taylor. Implementation and validation of the Fiala tire model in Chrono. Technical Report TR-2015-13: `http://sbel.wisc.edu/documents/TR-2016-06.pdf`, University of Wisconsin–Madison, 2015.

[29] M. Taylor and R. Serban. Validation of Basic Modeling Elements in Chrono. Technical Report TR-2015-05: `http://sbel.wisc.edu/documents/TR-2015-05.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2015.

[30] M. Taylor, R. Serban, and D. Negrut. Basic Comparison of Chrono::Vehicle and MSC ADAMS/Car. Technical Report TR-2016-15: `http://sbel.wisc.edu/documents/TR-2016-15.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[31] D. Negrut and R. Serban. Posing Multibody Dynamics with Friction and Contact as a Differential Algebraic Inclusion Problem. Technical Report TR-2016-12: `http://sbel.wisc.edu/documents/TR-2016-12.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[32] M. Kwarta and D. Negrut. Using the Complementarity and Penalty Methods for Solving Frictional Contact Problems in Chrono: Validation for the Cone Penetrometer Test. Technical Report TR-2016-16: `http://sbel.wisc.edu/documents/TR-2016-16.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[33] M. Kwarta and D. Negrut. Using the Complementarity and Penalty Methods for Solving Frictional Contact Problems in Chrono: Validation for the Triaxial Test. Technical Report TR-2016-17: `http://sbel.wisc.edu/documents/TR-2016-17.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[34] M. Kwarta and D. Negrut. Using the Complementarity and Penalty Methods for Solving Frictional Contact Problems in Chrono: Validation for the Shear-Test with Particle Image Velocimetry. Technical Report TR-2016-18: `http://sbel.wisc.edu/documents/TR-2016-18.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[35] A. Tasora, D. Mangoni, and D. Negrut. An Overview of the Chrono Soil Contact Model (SCM) Implementation. Technical Report TR-2017-01: `http://sbel.wisc.edu/documents/TR-2017-01.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2017.

[36] H. Mazhar, R. Serban, and D. Negrut. Using Chrono::FSI and Chrono::Vehicle for the Analysis of Sloshing Phenomena in Vehicle Dynamics Maneuvers. Technical Report TR-2017-02: `http://sbel.wisc.edu/documents/TR-2017-02.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2017.

[37] M. Rakhsha, A. Pazouki, and D. Negrut. Incompressible implicit SPH. Technical Report TR-2016-14: `http://sbel.wisc.edu/documents/TR-2016-14.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[38] B. Peterson and A. M. Recuero. Validation of a bilinear, gradient-deficient ancf shell element in Chrono::FEA – `http://www.projectchrono.org/assets/validations/fea/ancfshellvalidation.pdf`. Technical report, University of Wisconsin–Madison, 2016.

[39] A. M. Recuero and D. Negrut. Chrono support for ANCF finite elements: Formulation and validation aspects. Technical Report TR-2016-11: `http://sbel.wisc.edu/documents/TR-2016-11.pdf`, Simulation-Based Engineering Laboratory, University of Wisconsin-Madison, 2016.

[40] SBEL. Machine-Ground Interaction Consortium (MaGIC), Simulation-Based Engineering Laboratory, University of Wisconsin-Madison. `http://outreach.sbel.wisc.edu/Consortiums/`. Accessed: 2016-09-09.

[41] D. Negrut. Class Notes: High Performance Computing for Engineering Applications. `http://sbel.wisc.edu/Courses/ME964/`. Accessed: 2015-02-09.

[42] Project Chrono Tutorials. ProjectChrono tutorials web page. `http://outreach.sbel.wisc.edu/ChronoTutorials/`. Accessed: 2017-06-22.

[43] P. Cundall and O. Strack. A discrete element model for granular assemblies. *Geotechnique*, 29:47–65, 1979.

[44] Jean J. Moreau. Standard inelastic shocks and the dynamics of unilateral constraints. In G. Del Piero and F. Macieri, editors, *Unilateral Problems in Structural Analysis*, pages 173–221, New York, 1983. CISM Courses and Lectures no. 288, Springer–Verlag.

[45] D.E. Stewart. Convergence of a time-stepping scheme for rigid body dynamics and resolution of Painlevé's problems. *Archive Rational Mechanics and Analysis*, 145(3):215–260, 1998.

[46] T. Heyn, M. Anitescu, A. Tasora, and D. Negrut. Using Krylov subspace and spectral methods for solving complementarity problems in many-body contact dynamics simulation. *IJNME*, 95(7):541–561, 2013.

[47] M. Anitescu. Optimization-based simulation of nonsmooth rigid multibody dynamics. *Mathematical Programming*, 105(1):113–143, 2006.

[48] D. Negrut, R. Rampalli, G. Ottarsson, and A. Sajdak. On the use of the HHT method in the context of index 3 Differential Algebraic Equations of Multibody Dynamics. *ASME Journal of Computational and Nonlinear Dynamics*, 2, 2007.

[49] E. J. Haug. *Computer-Aided Kinematics and Dynamics of Mechanical Systems Volume-I.* Prentice-Hall, Englewood Cliffs, New Jersey, 1989.

[50] A. A. Shabana. *Computational Continuum Mechanics.* Cambridge University Press, first edition, 2008.

[51] Marco Schwarze and Stefanie Reese. A reduced integration solid-shell finite element based on the eas and the ans conceptgeometrically linear problems. *International Journal for Numerical Methods in Engineering*, 80(10):1322–1355, 2009.

[52] CA Felippa and B Haugen. A unified formulation of small-strain corotational finite elements: I. theory. *Computer Methods in Applied Mechanics and Engineering*, 194(21):2285–2335, 2005.

[53] Alexey Stomakhin, Craig Schroeder, Lawrence Chai, Joseph Teran, and Andrew Selle. A material point method for snow simulation. *ACM Transactions on Graphics (TOG)*, 32(4):102, 2013.

[54] Alexey Stomakhin, Craig Schroeder, Chenfanfu Jiang, Lawrence Chai, Joseph Teran, and Andrew Selle. Augmented mpm for phase-change and varied materials. *ACM Transactions on Graphics (TOG)*, 33(4):138, 2014.

[55] J. J. Monaghan. Smoothed Particle Hydrodynamics. *Reports on Progress in Physics*, 68(1):1703–1759, 2005.

[56] K. Bodin, C. Lacoursiere, and M. Servin. Constraint fluids. *IEEE Transactions on Visualization and Computer Graphics*, 18(3):516–526, 2012.

[57] E. Hairer, S. Norsett, and G. Wanner. *Solving Ordinary Differential Equations I: Nonstiff Problems.* Springer, Berlin, 2009.

[58] Project Chrono Development Team. Chrono: An Open Source Framework for the Physics-Based Simulation of Dynamic Systems. `https://github.com/projectchrono/chrono`. Accessed: 2017-05-07.

[59] Project Chrono. An Open Source Framework for the Physics-Based Simulation of Dynamic Systems. http://www.projectchrono.org, 2014.

[60] Project Chrono. ProjectChrono API Web Page. http://api.chrono.projectchrono.org/index.html. Accessed: 2016-03-07.

[61] Project Chrono Development Team. Chrono Reference Manual. http://api.chrono.projectchrono.org/refman.pdf. Accessed: 2016-09-11.

[62] Project Chrono. ProjectChrono tutorials web page. http://api.chrono.projectchrono.org/tutorial_root.html. Accessed: 2016-03-07.

[63] Chrono Development Team. Project Chrono user forum. https://groups.google.com/forum/#!forum/projectchrono. Accessed: 2016-09-11.

[64] Project Chrono. ProjectChrono stats. https://www.openhub.net/p/projectchrono/analyses/latest/languages_summary. Accessed: 2016-05-27.

[65] Project Chrono. Project Chrono Build Results. http://buildbot.sbel.org. Accessed: 2016-03-07.